Rule as the Sum of Rulelessness? Order, Chaos, and Patterns in Nature

David Feldman

19 February 1998

University of California, Davis
and
The Santa Fe Institute

dfeldman@landau.ucdavis.edu

http://leopard.ucdavis.edu/dave/

        the news to my left over the dunes
and reeds and bayberry clumps was
        fall: thousands of tree swallows
        gathering for flight:
        an order held
        in constant change: a congregation
rich with entropy: nevertheless, separable,
noticeable
    as one event,
            not chaos: preparations for
flight from winter,
cheet, cheet, cheet, cheet, wings rifling
the green clumps,
beaks
at the bayberries
    a perception full of wink, flight,
    curve, sound:
    the possibility of rule as the sum of
rulelessness:
the "field" of action
with moving, incalculable center:

A. R. Ammons
in *Corsons's Inlet* (1965)

Ammons' Observations:

- Flock is neither ordered nor chaotic.
- *Rule as the sum of rulelessness:*

  — Short-range interactions produce large-scale organization.

  — Simple rules produce complicated, complex patterns.

Questions:

- What mathematical vocabulary can we use to describe phenomena like the flock of swallows?
- How can we discover and characterize patterns? Is there an objective notion of pattern, or is the discovery of patterns a purely subjective process?
- How does organization arise in nature? How do chaos and order combine?

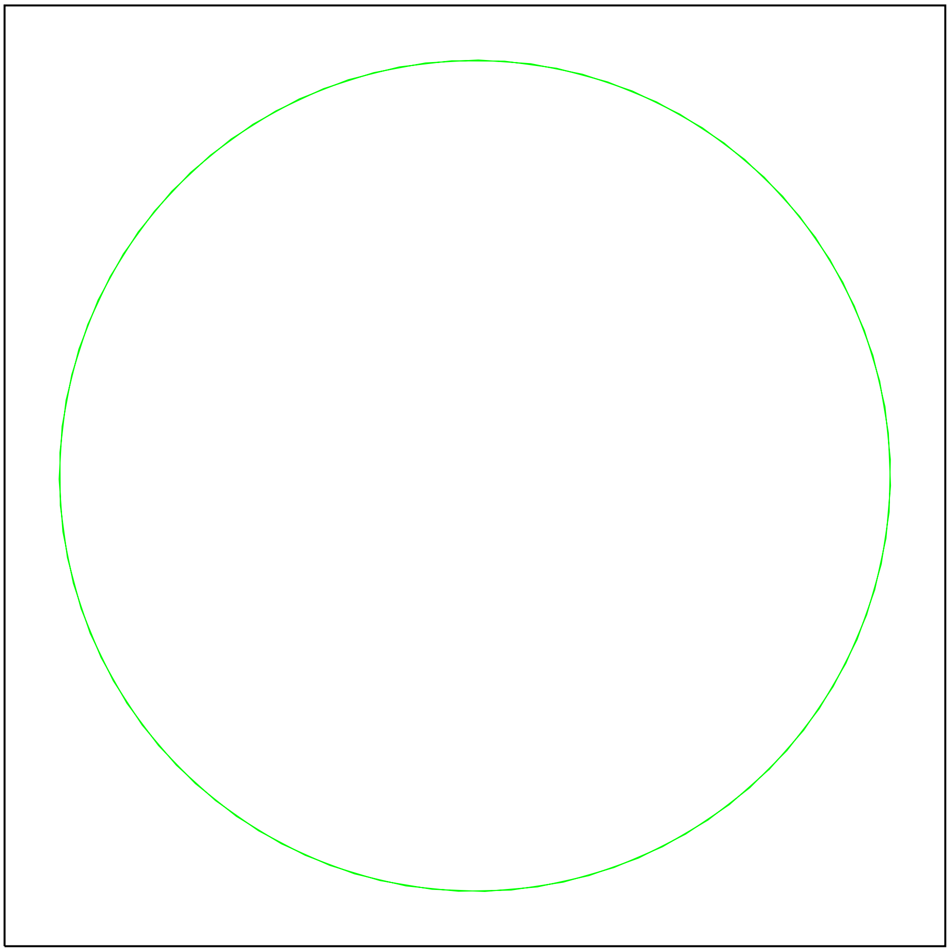
Outline

1. **Introduction.**
2. **Lazy Baker Map:** Examples of chaos and patterns.
3. **Computation Theory:** A method for describing patterns.
4. **Information Theory:** A method for measuring unpredictability and memory.
5. **Conclusion.**

Dynamical Systems

Mathematical system that evolves forward in time.

Example: Moon orbits earth under the influence of earth's gravity:



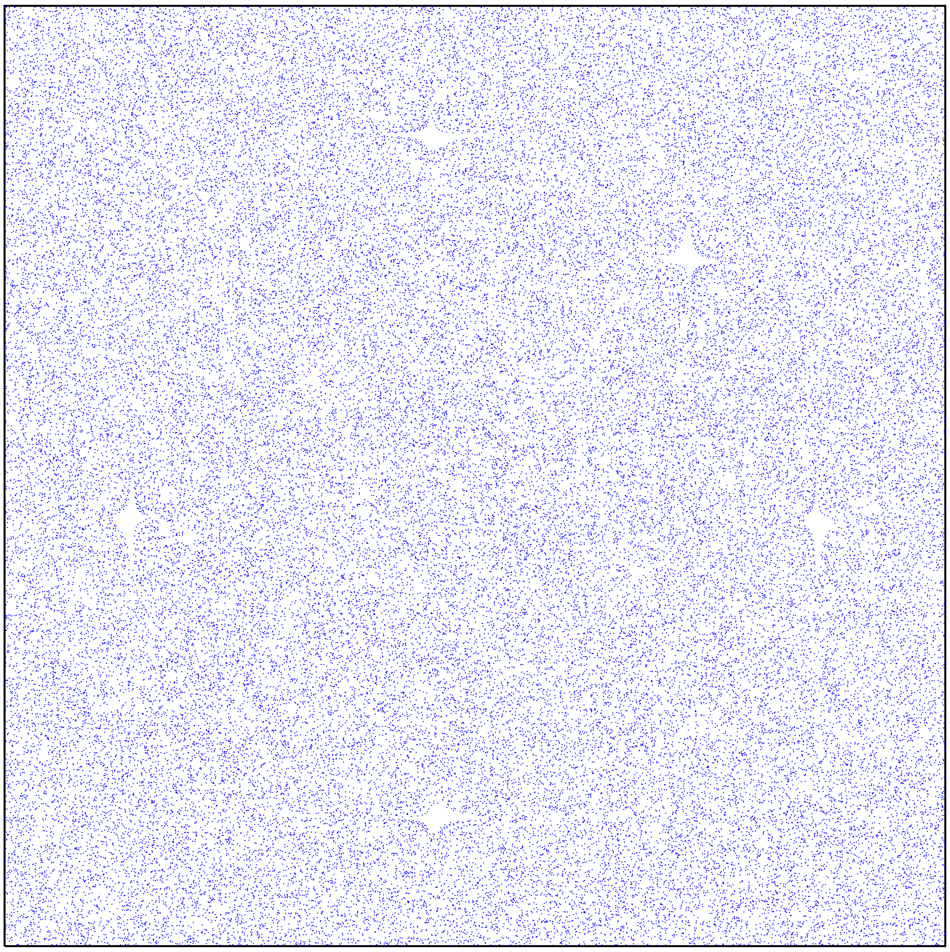
The moon follows a very regular, predictable path.

Another Dynamical System:

An annoying dripping faucet.

The sequence of the drops is irregular and very difficult to predict.

The Lazy Baker Map

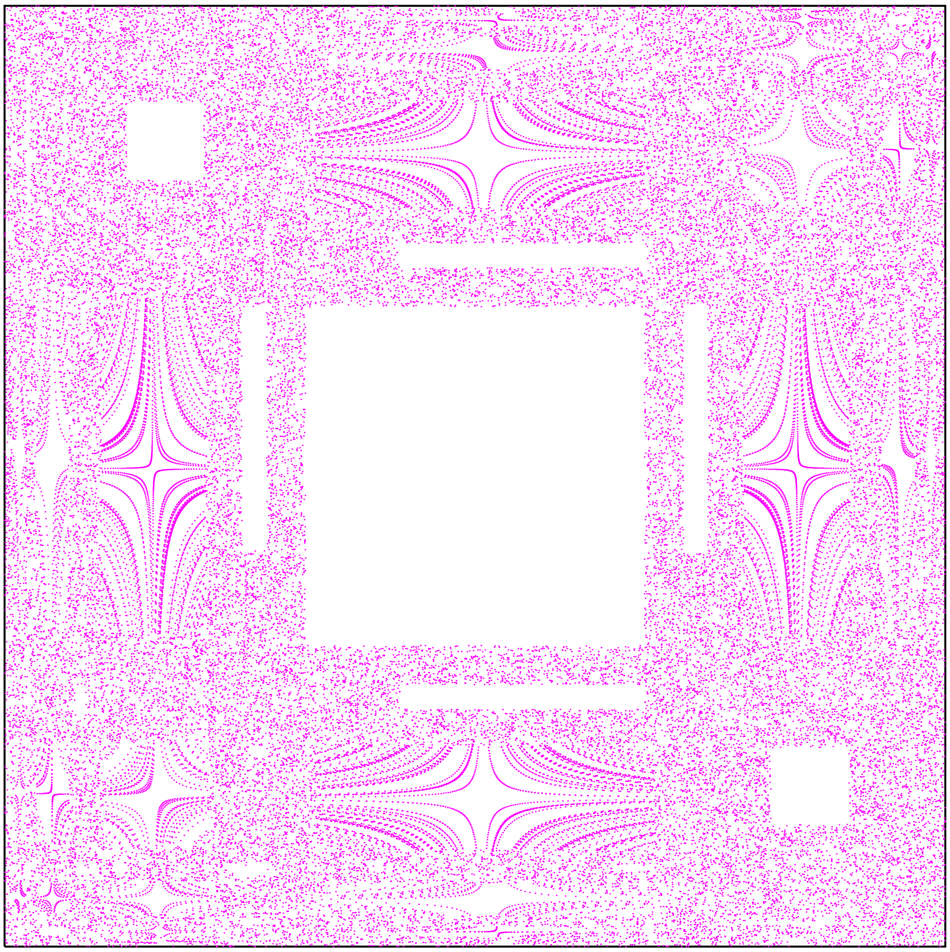


The path taken by the raisin appears random.

“Butterfly Effect.”

The Lazy Baker Map

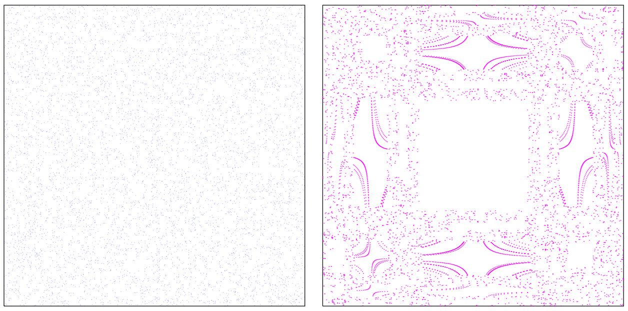
Different Parameters



The path taken by the raisin traces out an intricate pattern—rule as the sum of rulelessness!

Morals of the Lazy Baker Story

- **Very** simple programs produce very complicated behavior. Both random and patterned:



- Tiny changes in the program make large differences in the overall behavior.

- Focusing on the “rules”—the equations—is not the most revealing analysis for many dynamical systems. Since behavior depends sensitively on the details of the “rules”, guessing the “rules” for an observed phenomena is very difficult.

Can't Guess the Rules

Traditional Approach:

Make observations $\Rightarrow$ Guess equations.

Example: Brahe and Kepler observe planetary motion.
Newton guesses law of gravitation.

Alternative Approaches:

If observed phenomena are noisy or irregular, very difficult to guess equations. What to do???

1. Statistics
2. **Computation Theory**
3. **Information Theory**

Computation Theory

Idea: Classify sequences by considering how hard it is to recognize them.

Example 1: Accept all sequences of 1's and 0's except for those which have two or more 0's in a row.

$$1110101101$$

$$1101101001$$

This example is pretty easy. You don't need to remember much of the sequence as you scan it to figure out if it's ok.

Example 2: Accept all sequences with balanced parenthesis.

$$(\; (\;) \; (\;) \; (\;) \;) \; (\; (\;) \;)$$

$$(\; (\;) \; (\; (\;) \; (\;) \; (\;) \; (\;)$$

This example is harder.

Example 3: Accept all sequences which consist of any number of 1‘s, and only a **prime** number of consecutive 0’s!

$$11011001$$

$$11000011$$

$$11010000000000001$$

$$11 \quad \overbrace{00\cdots00}^{1031\ \text{elements}} \quad 11$$

We can make this notion of difficulty precise by considering the type of “machine” you would have to build to perform this recognition task.

Example 1: Accept all sequences of 1’s and 0’s except for

those which have two or more 0’s in a row.

$$1101101001$$

Sequence is OK if there exists a path through this machine as one scans left to right. Machine works for an arbitrarily long sequence.

Example 2: Accept all sequences with balanced

parenthesis.

$$(( ) (( ) ( ) ( ) ( )$$

To check the sequences you need to build a device with memory: a **Stack**.

Put a plate on the stack if you see a left paren $($ and take a plate off the stack if you see a right paren $)$.

If there are no plates left on the stack at the end of scanning the sequence, then sequence is ok.

Example 3: Accept all sequences which consist on any

number of 1's, and only a **prime** number of consecutive 0's.

$$11 \quad \overbrace{00 \cdots 00}^{1031 \text{ elements}} \quad 11$$

For this task you'll need to build a machine that can perform division: desktop computer or a calculator.

Summary of Computation Theory

Classify sequences by considering how hard it is to recognize them.

Generalizing vastly:

- Most sequences tend to fall into the same category as example 1. Example: The paths the raisin takes in the first lazy baker example.
- At the transition point between ordered and chaotic behavior, the sequences become harder to describe and more complex machines are necessary. Example: paths taken by the raisin in the second example. (?)

Information Theory

The Shannon Entropy H **measures how uncertain an uncertain event is:**

$$H[X] \equiv \sum_x -\Pr(x) \log_2 \Pr(x) .$$

Examples:

- A **Fair Coin**: (Probability of heads $= \frac{1}{2}$) has an unpredictability of 1.

- A **Biased Coin**: (Probability of heads $= 0.9$) has an unpredictability of 0.47.

Mutual Information

The **Mutual Information** of two random events X and Y is defined by:

$$I[X;Y] \equiv \text{Uncertainty of } X$$

$$- \text{Average Uncertainty of} X \text{ after } Y \text{ is Known} .$$

Mutual Information Example

What's for dinner at the cafeteria depends on what was for lunch. If cauliflower for lunch, more likely that cauliflower casserole will be for dinner.

If I'm wondering what's for dinner, and you tell me that there was cauliflower for lunch, I would thank you for the information you just gave me.

- Note that we can calculate the lunch-dinner mutual information without knowing the "laws" of the cafeteria.

- The mutual information vanishes if the cafeteria is perfectly random or perfectly predictable.


Information Theory Summary:

- The Shannon entropy provides a way to measure the **unpredictability** of a dynamical system.

  — How unpredictable is the future behavior of the flock of birds, or of the raisin in the dough?

- Mutual information can be used to measure of the **memory** or **correlations** in a dynamical system.

  — How much information does knowing the past behavior of the flock give me about the future behavior?

  — If I know the position of the left half of the flock, how much does that tell me about the right half of the flock?

My Work

- Interested in ways in which order and randomness combine to produce patterns
- Interested in systems in which “Rule is the Sum of Rulelessness.”
- Use computation theory and information theory to analyze such systems.
- I think that computation theory and information theory complement each other well, and can greatly enrich the vocabulary we use to describe complex systems.

Conclusions

- What mathematical vocabulary can we use to describe phenomena like the flock of swallows?

- How can we discover and characterize patterns? Is there an objective notion of pattern, or is the discovery of patterns a purely subjective process?

  — Guessing the rules usually not possible.

  — Use Information theory and computation theory instead.

- How does organization arise in nature? How do chaos and order combine?

  — Beats me! I suspect that there are few, if any, general principles of organization.

  — However, “Rule as the Sum of Rulelessness” often holds.

Order is not sufficient. What is required, is something much more complex. It is order entering upon novelty; so that the massiveness of order does not denigrate into mere repetition; and so that the novelty is always reflected upon a background of system. . . .

Some new principle of refreshment is required. The art of progress is to preserve order amid change, and to preserve change amid order. Life refuses to be embalmed alive.

A. N. Whitehead

Process and Reality **(1929)**