Clarifying Notes on Section 27.5: CA Naming Conventions

Chaos and Fractals: An Elementary Introduction

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The convention for naming elementary CAs is entirely standard. In Section 27.5 I follow this convention. So, for example, the CA that I call 30 is the same CA that everyone else would call 30.

However, I arrive at the conventional name by an unconventional means, and this could possibly cause some confusion. I think it will be most clear to illustrate this with an example. First, I will show my way of setting up the equation that leads to a CA name, and then I will show the way this is more typically done.

Method from Section 27.5

Figure 1: CA Rule 30, shown as it would be in the text. Note that the neighborhoods are listed, left-to-right, from the “whitest” to the “blackest.”

Consider the CA shown in Fig. 1. Eight three-site neighborhood are listed, and for each neighborhood the output symbol is shown. These output symbols are shown again in Eq. (1):

$$\square\blacksquare\blacksquare\blacksquare\blacksquare\square\square\square\ . \tag{1}$$

Or, using 0 for $\square$ and 1 for $\blacksquare$, one writes:

$$0\,1\,1\,1\,1\,0\,0\,0\,. \tag{2}$$

This sequence of 0's and 1's is then converted into a base-10 number by viewing the sequence as a binary number, *left-to-right*:

$$\begin{aligned}
0\,1\,1\,1\,1\,0\,0\,0 \;&=\; (0\times 2^0) + (1\times 2^1) + (1\times 2^2) \\
&\quad + (1\times 2^3) + (1\times 2^4) + (0\times 2^5) \\
&\quad + (0\times 2^6) + (0\times 2^7) && (3)\\
&= 0+2+4+8+16+0+0+0 && (4)\\
&= 30\ . && (5)
\end{aligned}$$

Thus, the rule shown in Fig. 1 is rule 30.

Standard Method

The approach described above correctly leads to designating the CA of Fig. 1 as rule 30. However, the path taken to do so is not used by most other authors. This section illustrates the more standard approach. The CA of Fig. 1 can also written as shown in Fig. 2. Both figures show eight neighborhoods, but the order in which these neighborhoods are opposite. But both figures show the same rule. Each neighborhood's output is the same in both figures—the neighborhoods just appear in a different location in the list. For a given initial condition, the rules shown in Figs. 1 and 2 would produce the exact same space-time diagram, because the rules are exactly the same.

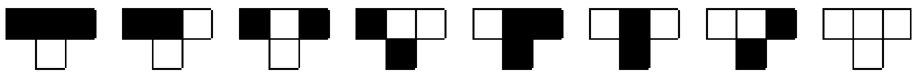

Figure 2: CA Rule 30, shown in the standard orientation. Note that the neighborhoods are listed, left-to-right, from the "blackest" to the "whitest."

The outputs for the rule shown in Fig. 2 are:

$$\square\square\square\blacksquare\blacksquare\blacksquare\blacksquare\square\,. \tag{6}$$

We now convert this to a binary number and then to base-10, but this time we read *right-to-left*. That is, the smallest binary digit is on the right, not the left:

$$\begin{aligned} 0\,0\,0\,1\,1\,1\,1\,0 \;&=\; (0\times 2^7) + (0\times 2^6) + (0\times 2^5) \\ &\quad + (1\times 2^4) + (1\times 2^3) + (1\times 2^2) \\ &\quad + (1\times 2^1) + (0\times 2^0) && (7) \\ &= 0+0+0+16+8+4+2+0 && (8) \\ &= 30\,. && (9) \end{aligned}$$

Thus, the rule shown in Fig. 2 is also rule 30.

Conclusion

Both of the methods described above are correct, in that they yield the standard name for a given rule. However, the method referred to above as the "standard method" is much more standard. Almost any other reference on CAs will use this approach. I think the standard method is probably better than my method, since in the standard method the binary digits are read off in their usual order, with digits increasing right to left.

I don't know why I chose the left-to-right method that I wrote in the text. I suppose at the time it seemed natural to me to do it this way, and I didn't compare my discussion with any of the standard texts; I just checked to make sure that it gave the right rule, which is does.