Predictable Unpredictability: Strange Attractors and the Butterfly Effect

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Dynamical Systems

- Study of mathematical systems that change over time
- Also known as “Chaos Theory”
- Has led to rethinking of some fundamental categories in science

Outline:

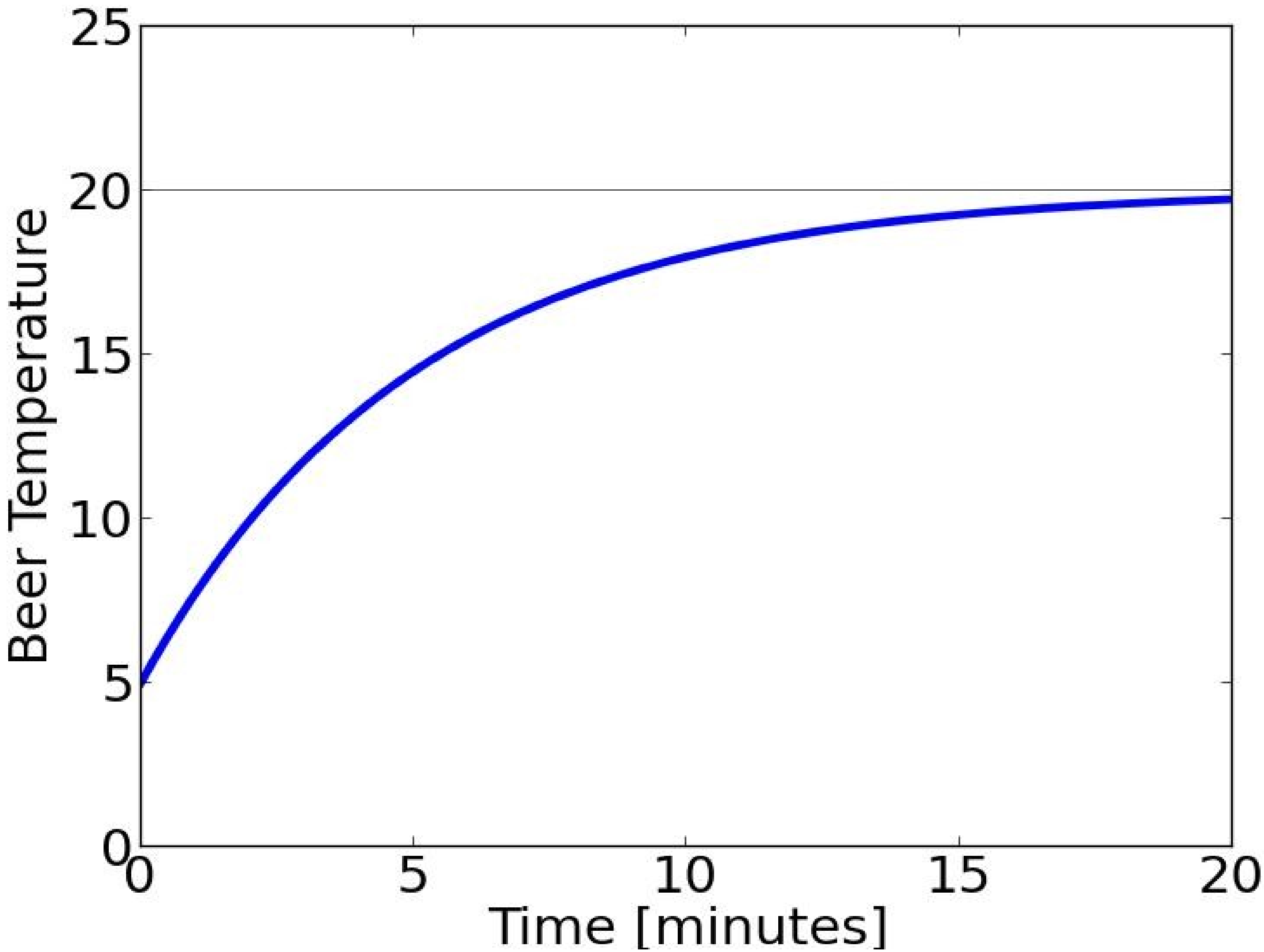
1. A Non-Chaotic Warm-up Example
2. Butterfly Effect
3. Strange Attractor
4. Conclusion

There will be lots of time for discussion and comments throughout the presentation

An Initial Example

- Newton's Law of Cooling describes the temperature of a glass of beer as it warms to room temperature
- $$\frac{dT}{dt} = 0.2(20 - T)$$
- This is a differential equation. It describes how temperature T changes with time t.
- The solution to this equation is T(t), the temperature as a function of time.

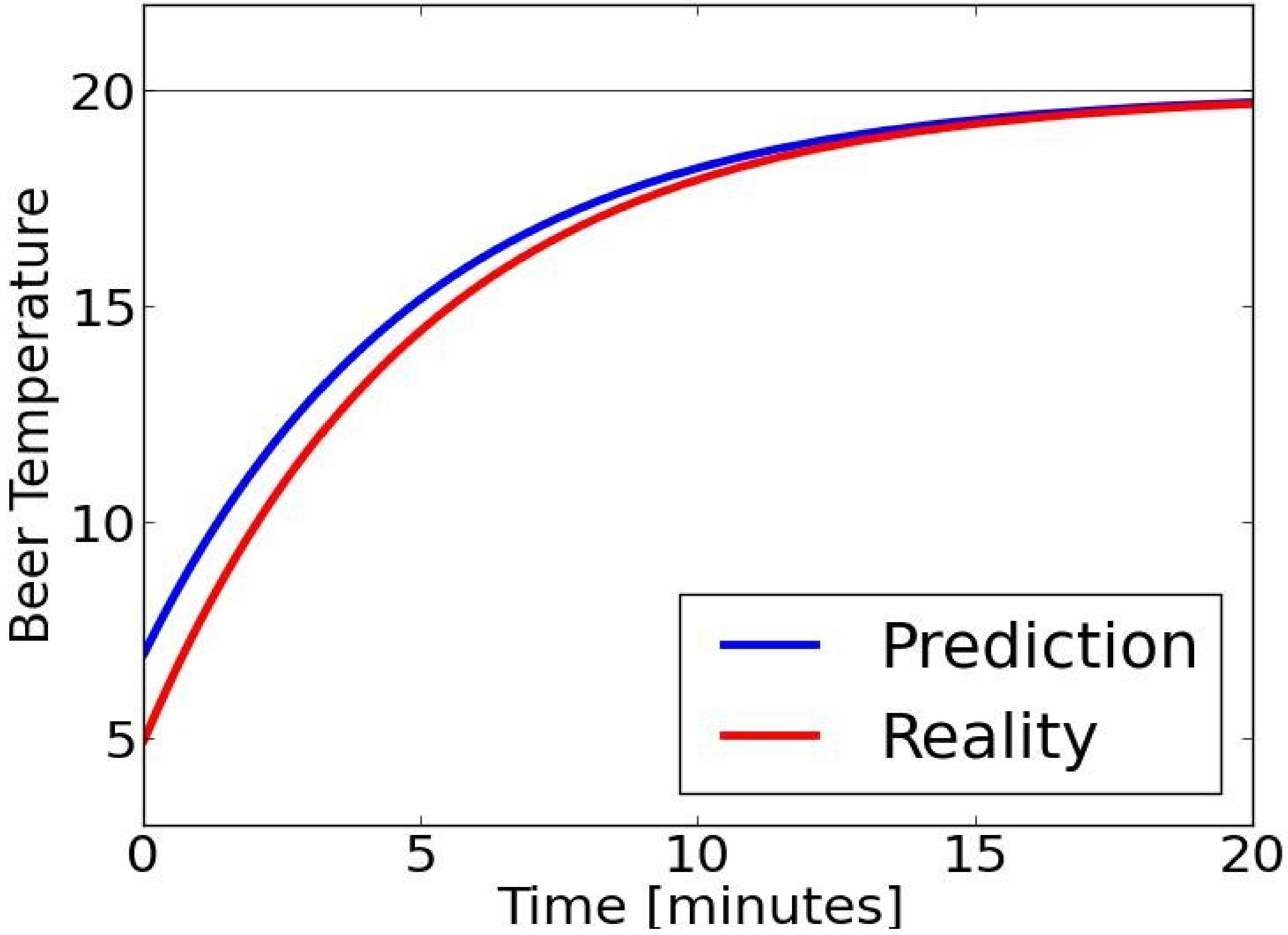
The Beer Warms Up



Predicting Beer Temperature

- We can use the equation to predict the temperature of the beer at a later time.
- We might be slightly wrong in our initial temperature measurement, but it won't matter much.

Predicting Beer Temperature



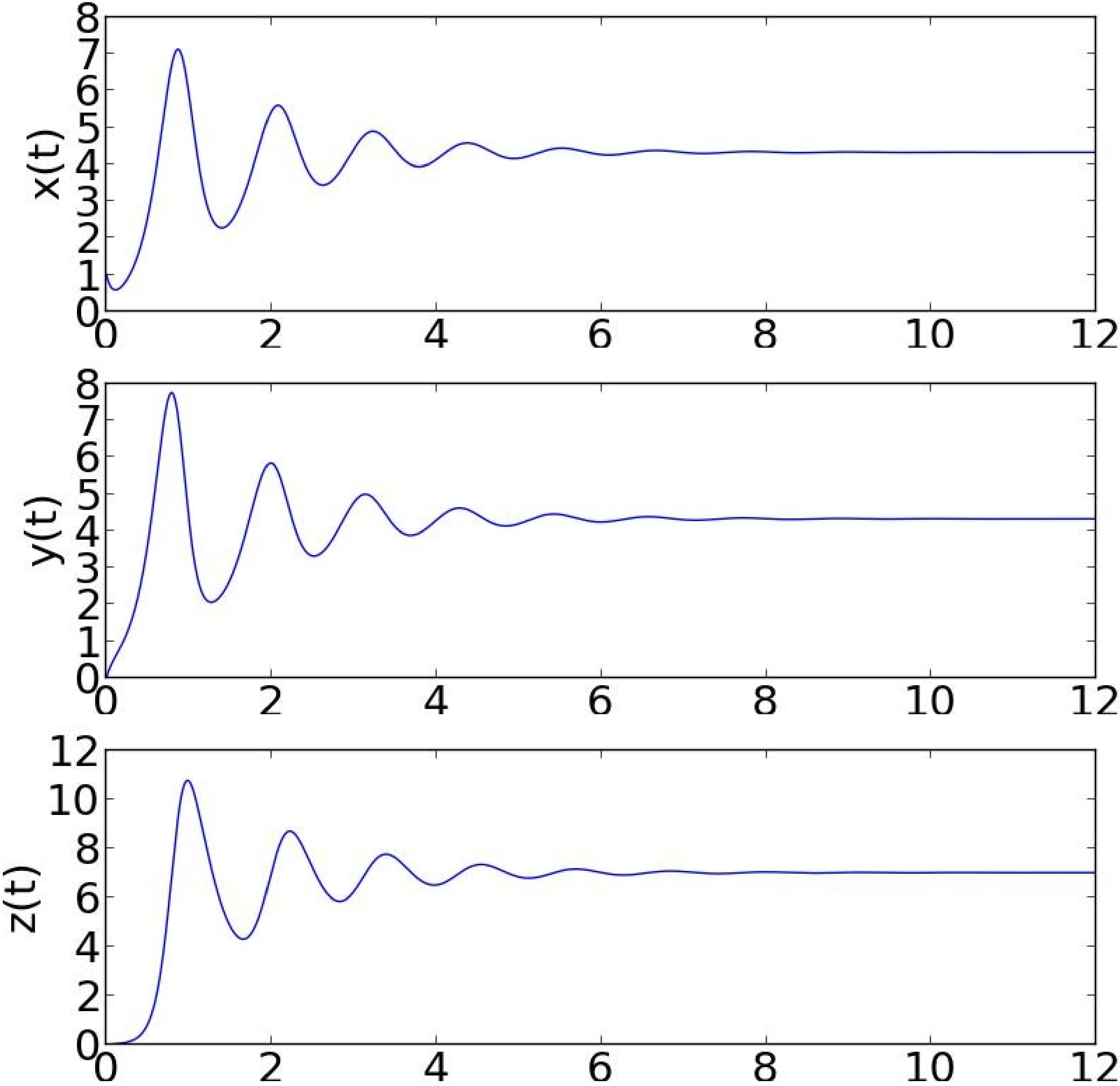
Laplace's Demon

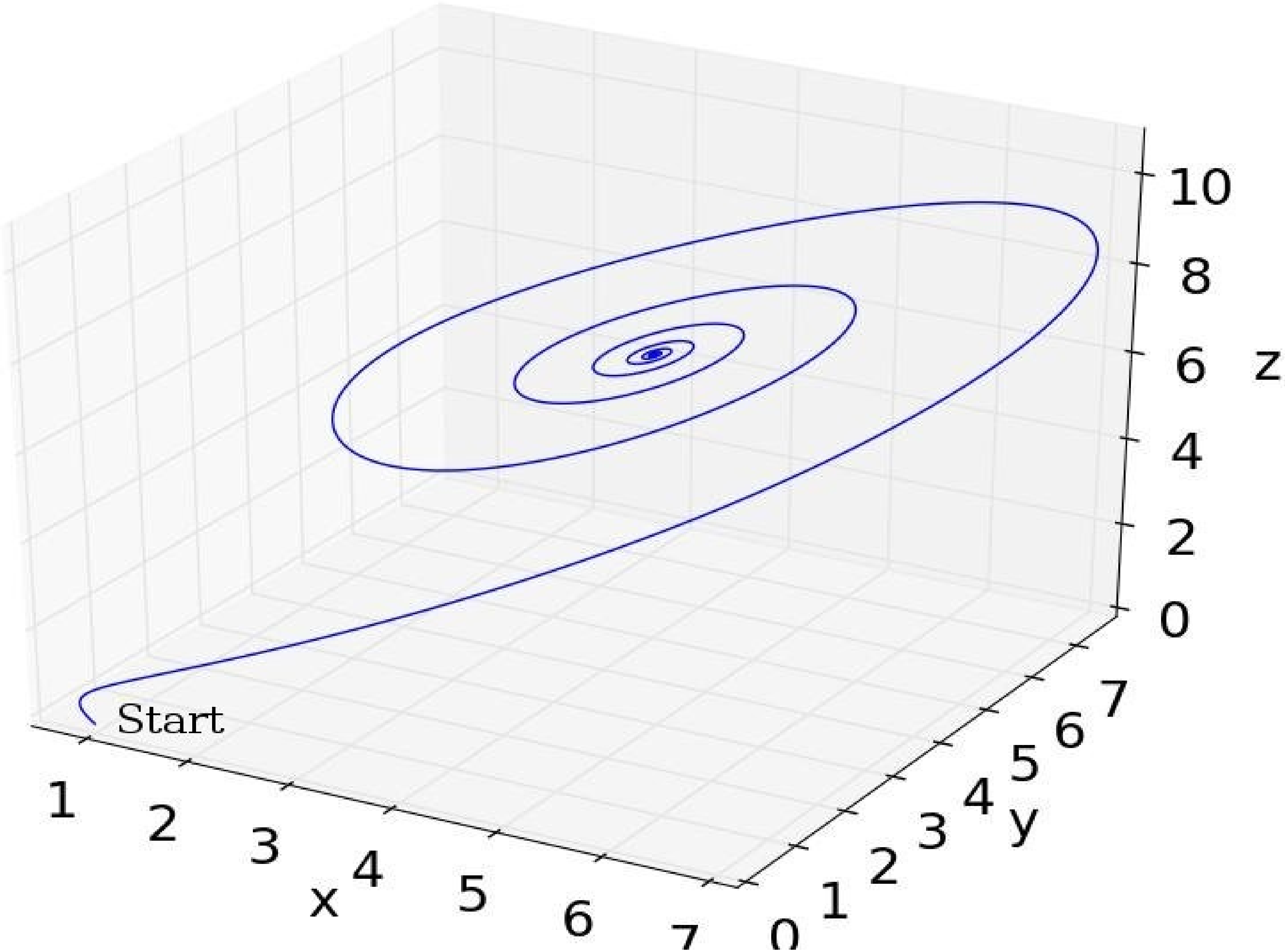
An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

(Pierre-Simon Laplace, 1814)

Another Example

- Here is another equation to study
- $$\frac{dx}{dt} = 10(y - x)\ , \quad \frac{dy}{dt} = x(8 - z) - y\ , \quad \frac{dz}{dt} = xy - \frac{8}{3}z$$
- Three equations, one each for x, y, and z.
- A very simple model of atmospheric convection

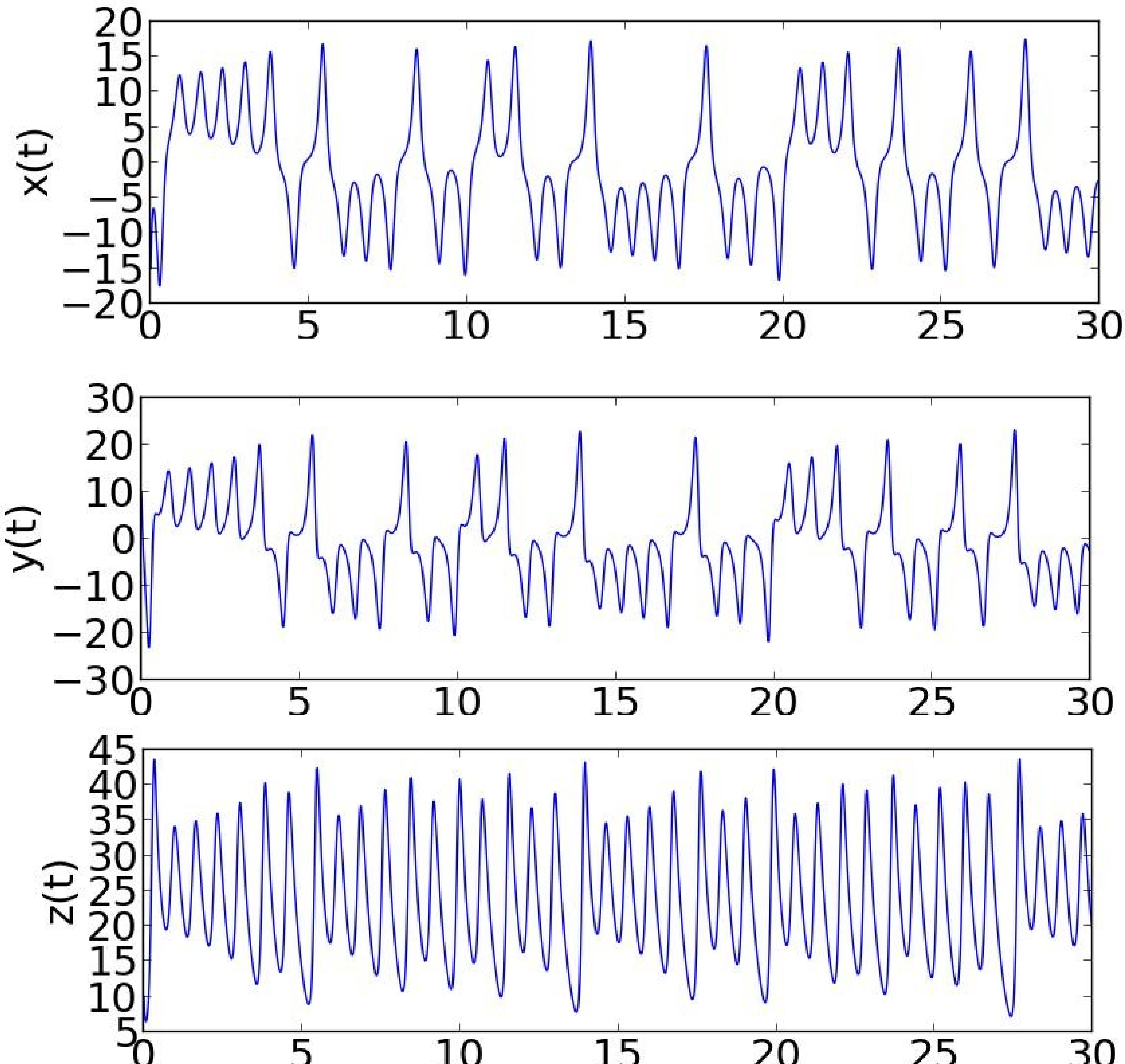

x(t)
0
1
2
3
4
5
6
7
8
0
2
4
6
8
10
12
y(t)
0
1
2
3
4
5
6
7
8
0
2
4
6
8
10
12
z(t)
0
2
4
6
8
10
12
0
2
4
6
8
10
12



Start
1
2
3
4
5
6
7
x
0
1
2
3
4
5
6
7
y
0
2
4
6
8
10
z

Another Example

- Let's change this equation slightly:
- $$\frac{dx}{dt} = 10(y - x)\ , \quad \frac{dy}{dt} = x(28 - z) - y\ , \quad \frac{dz}{dt} = xy - \frac{8}{3}z$$
- Three equations, one each for x, y, and z.
- A very simple model of atmospheric convection

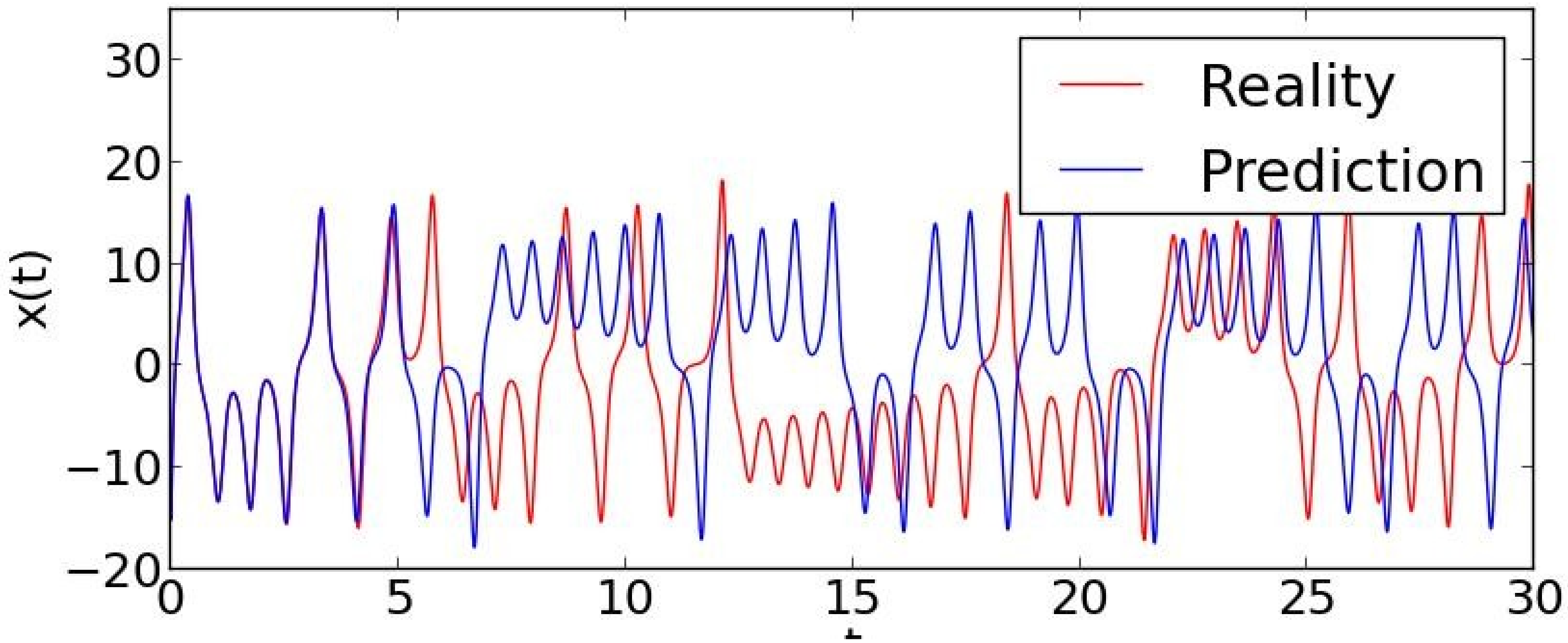

x(t)
y(t)
z(t)
20
15
10
5
0
−5
−10
−15
−20
30
20
10
0
−10
−20
−30
45
40
35
30
25
20
15
10
5
0
5
10
15
20
25
30

Prediction with the Lorenz Equations

The initial measurement is slightly off:

Model: x=15.1. Reality: x=15.0

We can predict until around t=5

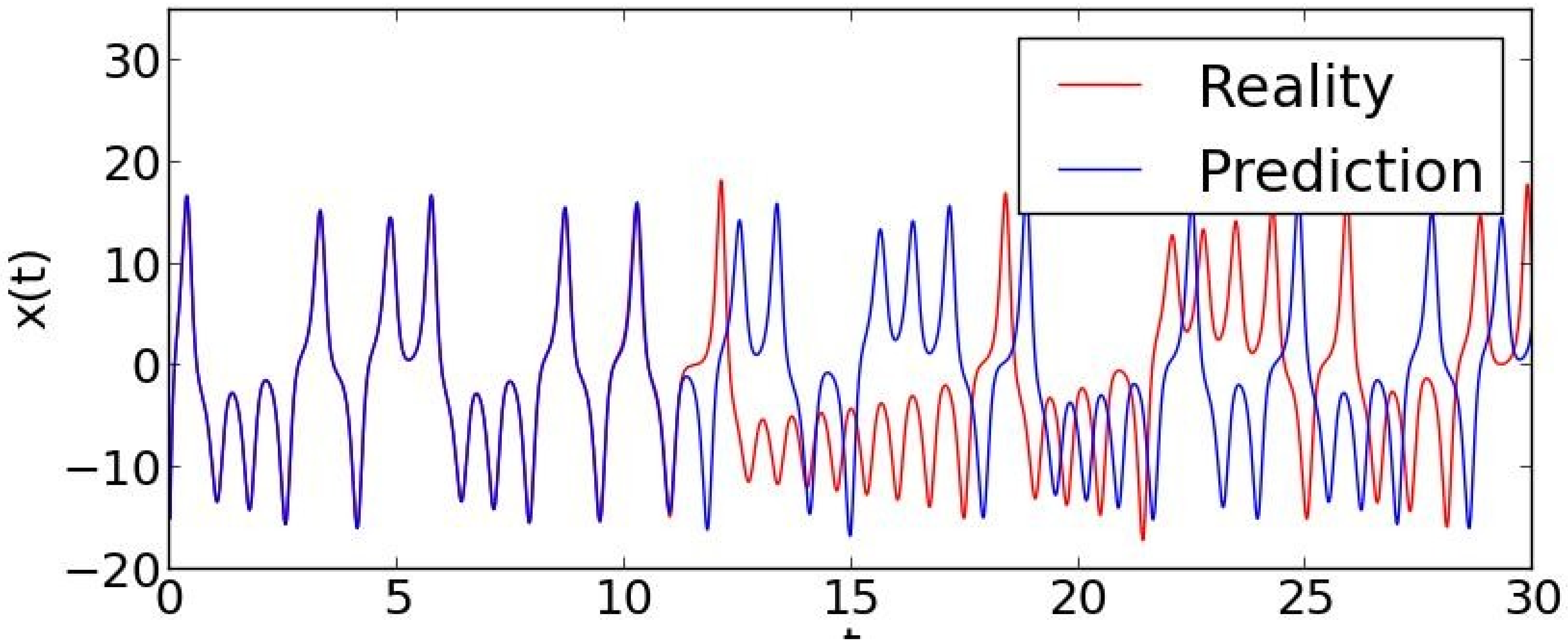


Prediction with the Lorenz Equations

The initial measurement is **much** better:

Model: x=15.0001. Reality: x=15.0

We can now predict until around t=11

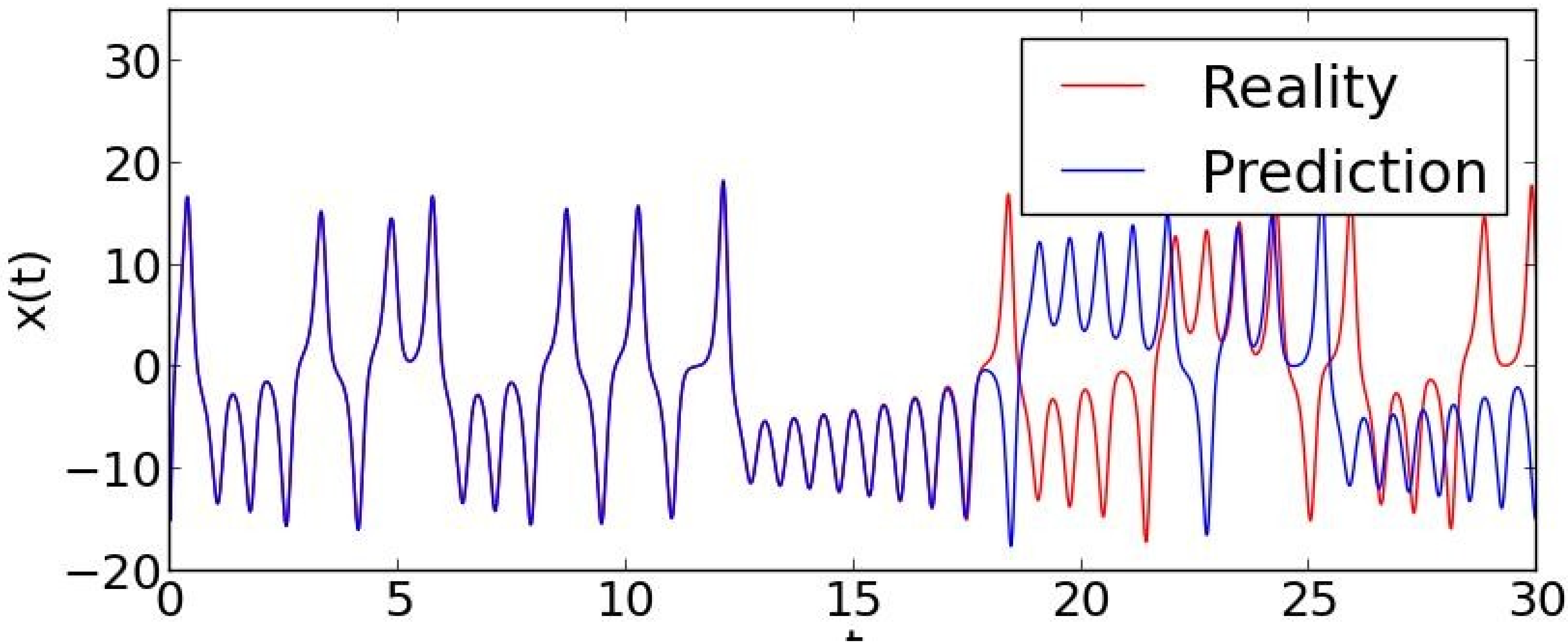


Prediction with the Lorenz Equations

The initial measurement is **amazing**:

Model: x=15.00000001. Reality: x=15.0

We can now predict until around t=18.



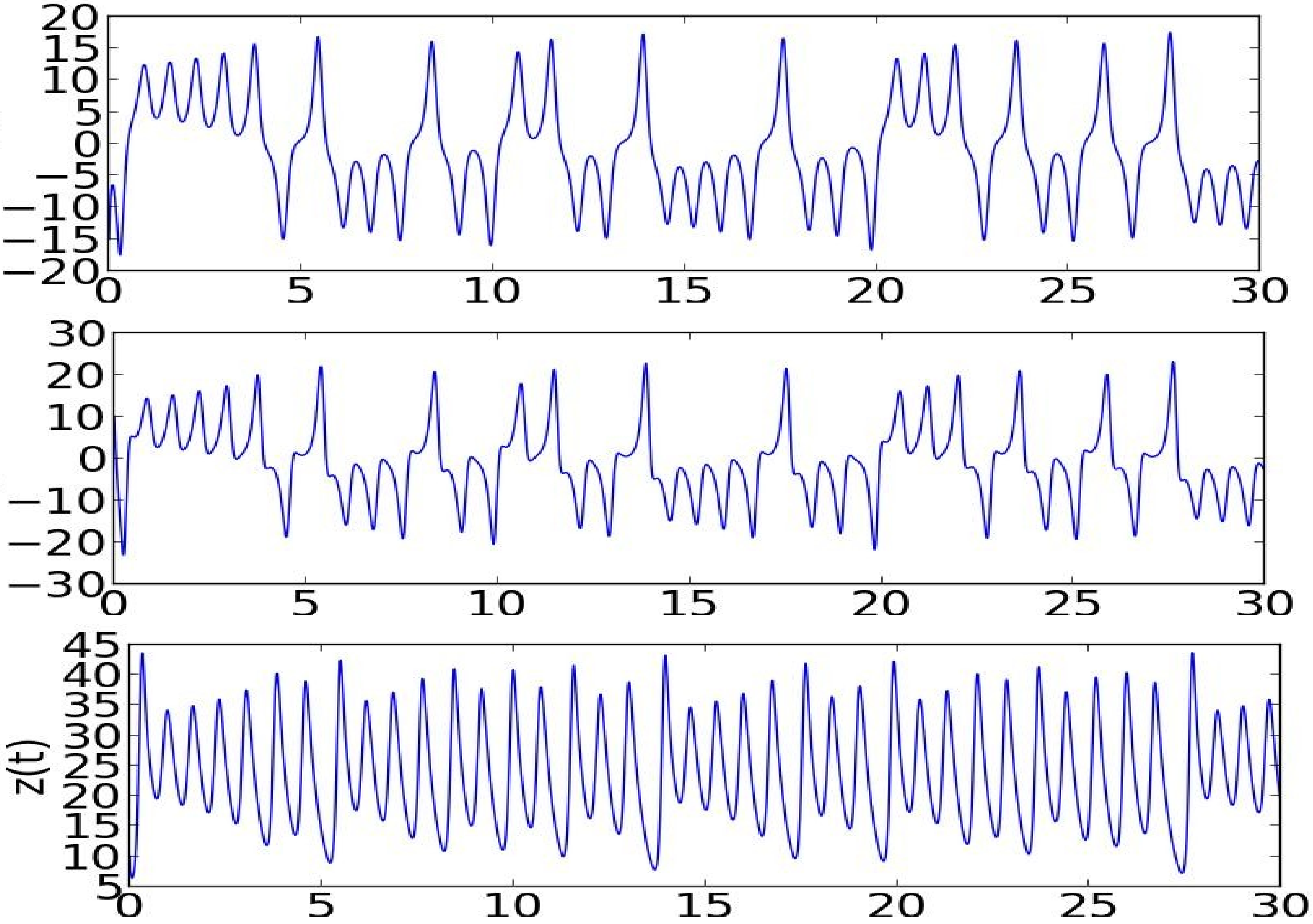
Butterfly Effect

- This is the Butterfly Effect
- A very small error in the initial condition grows extremely rapidly
- Long-term prediction is impossible
- A deterministic (rule-based) system can behave unpredictably
- Predictability causes unpredictability

Chaos!

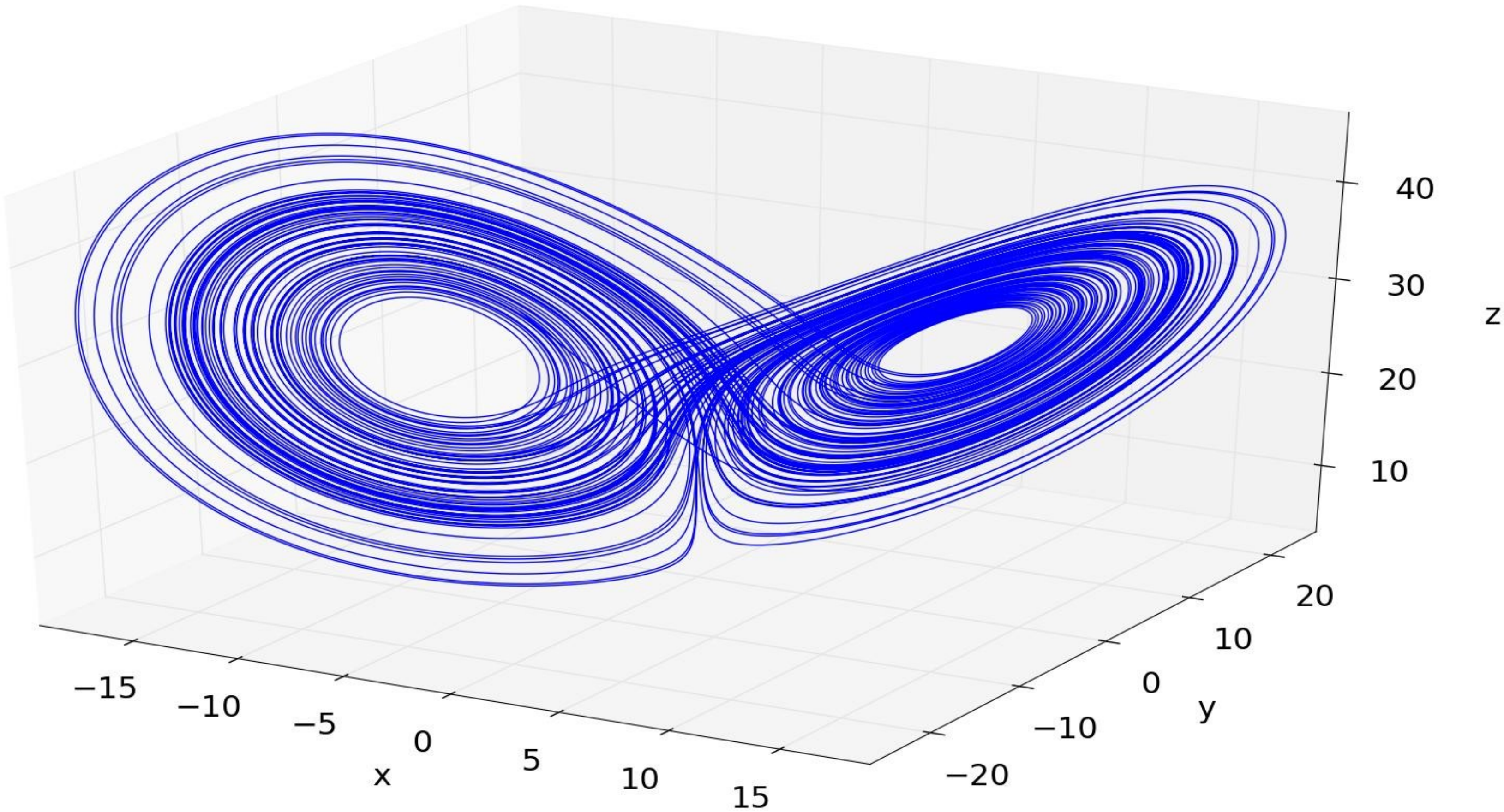
- A dynamical system is chaotic if:
  - It is deterministic
  - Solutions are bounded
  - Solutions are aperiodic (non-repeating)
  - Solutions have the butterfly effect
- Simple systems can be complicated and unpredictable
- Makes connections to many different fields

Chaos?



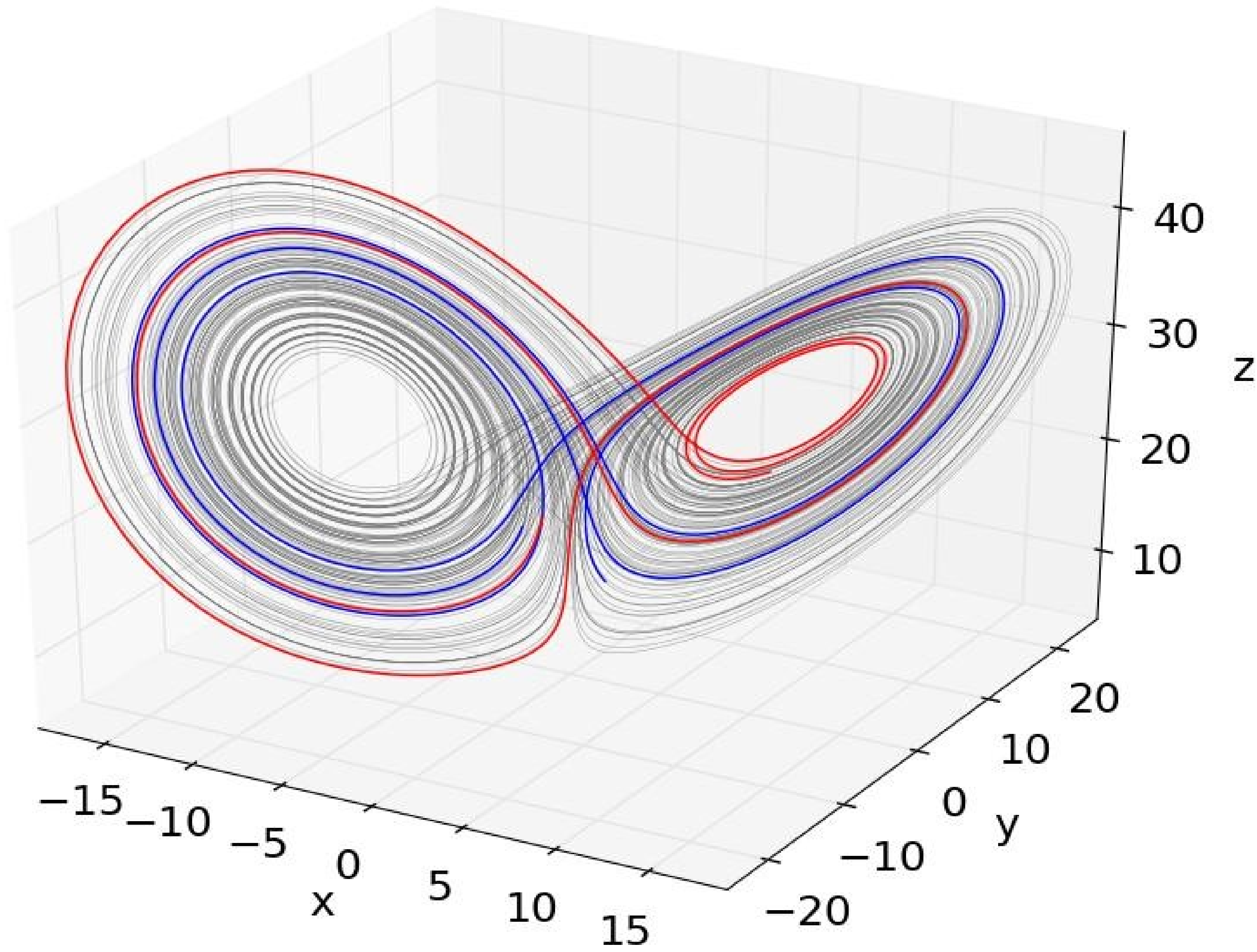
What happens if we plot x vs. y vs z?

A Strange Attractor



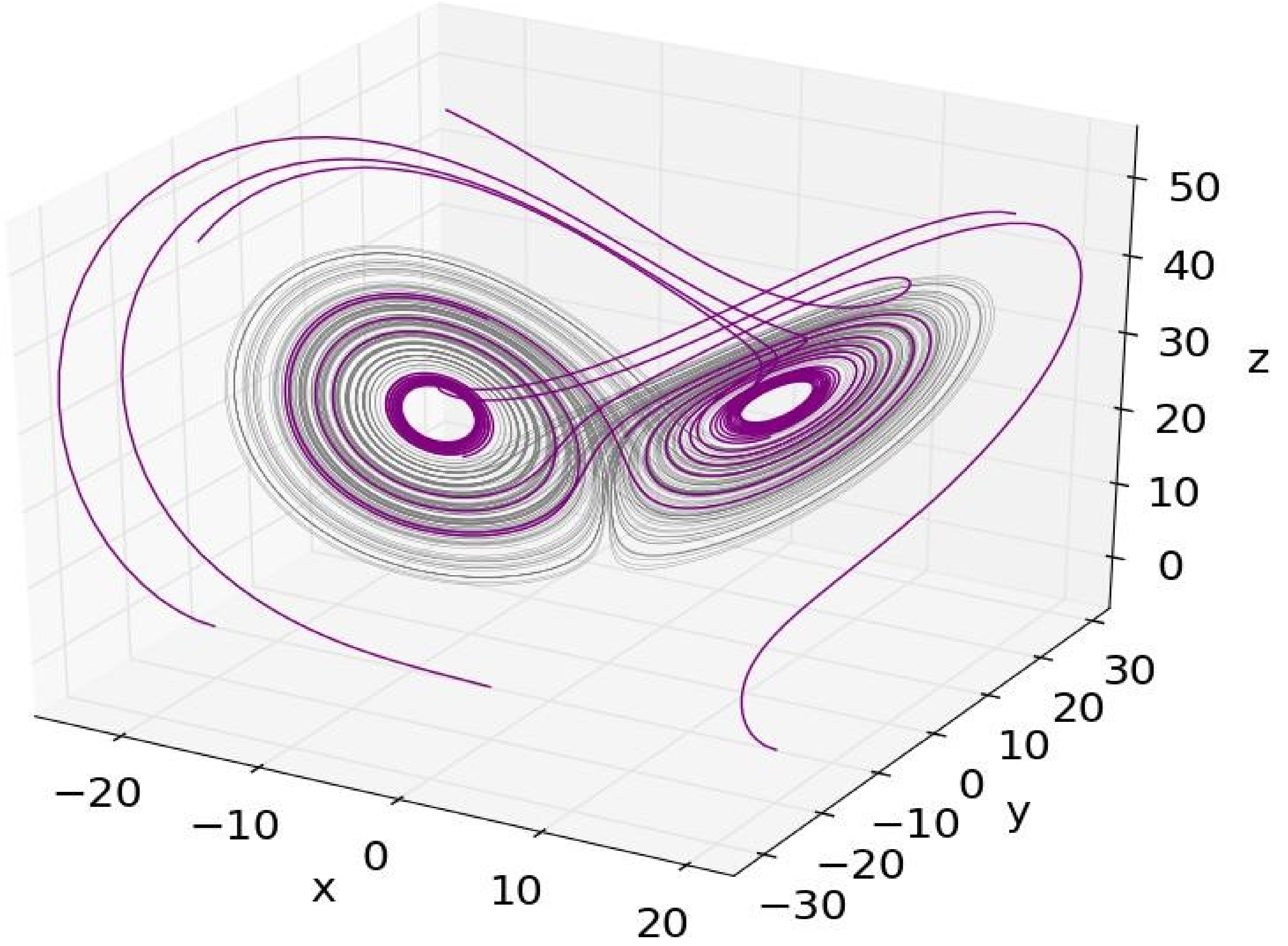
- We see a complex relationship among x, y, and z.
- The trajectory weaves through space but never repeats.

A Strange Attractor



- The motion on the attractor is chaotic

A Strange Attractor

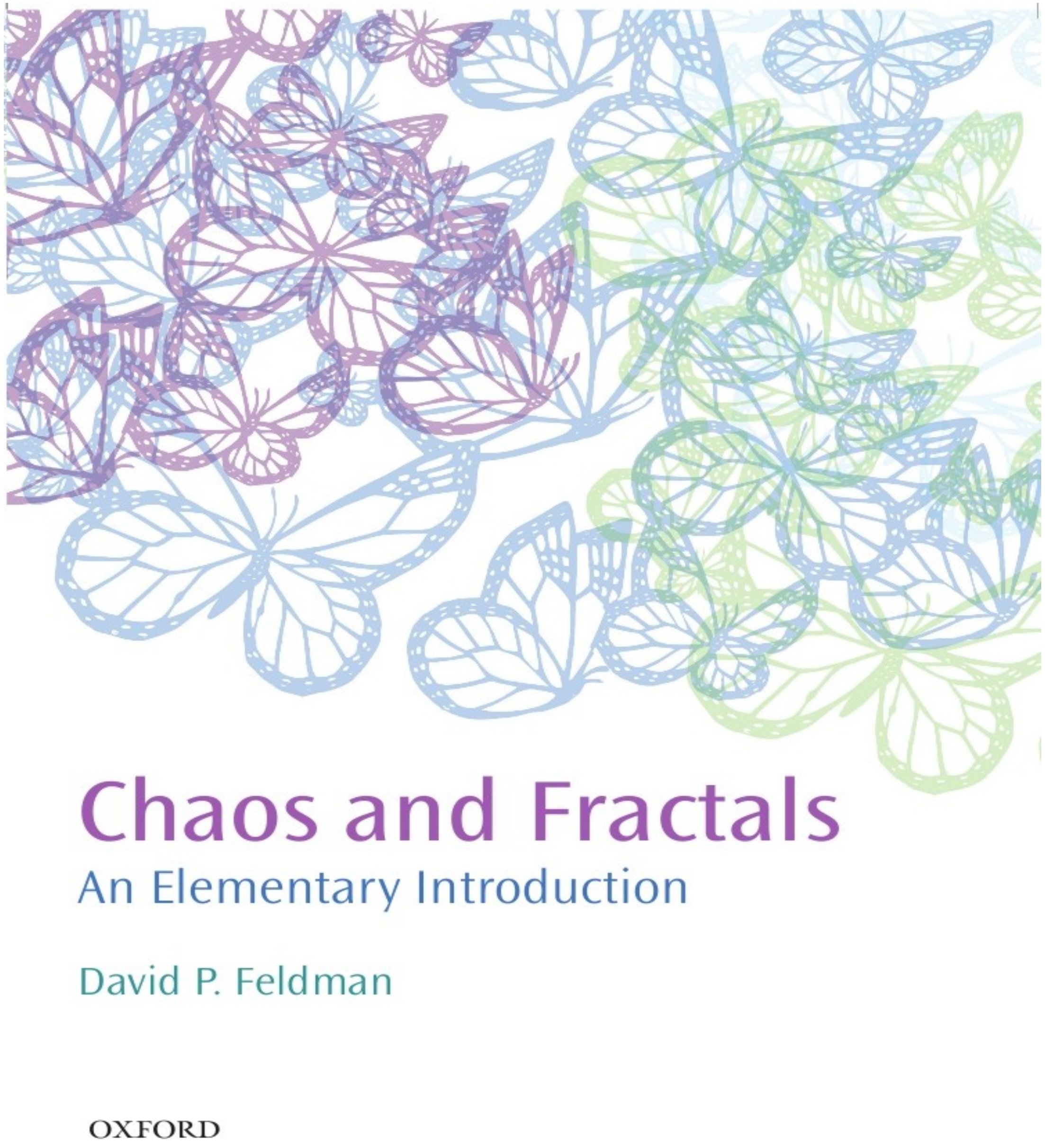


- But it attracts: all solutions are pulled toward it.

Strange Attractors & Chaos

- Locally Unpredictable, Globally Predictable
- Combine order and disorder
- They are “predictably unpredictable”
- Deterministic systems appear random
-

A New Textbook



Chaos and Fractals: An Elementary Introduction.

David P. Feldman. Oxford University Press. 2012.

http://chaos.coa.edu

A MOOC!

- I will teach a MOOC on Chaos and Dynamical Systems
- MOOC = Massive Open Online Course
- http://www.complexityexplorer.org
- January 6 – February 28, 2014
- Enrollment is free
- Only pre-requisite is high-school algebra

COMPLEXITY EXPLORER

Thank you