

Calculus I Midterm

October 10–12, 2006

Important Directions

- **You may not collaborate on this exam; do not work with others.**
- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.
- This exam is untimed, but unless I hear to the contrary, I expect you to finish by 8:00am on Friday, October 13.
- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.
- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.
- To receive full credit on most of these problems you must show your work clearly.

1. Short Answers. No explanations needed.
 - (a) Which function dominates as $x \rightarrow \infty$: x^3 or $900x$?
 - (b) Which function dominates as $x \rightarrow \infty$: $x^{1/9}$ or x^{-9} ?
 - (c) What is the period of $g(x) = 3 \cos(2x) + 4$?
2. Let $f(x) = 6x^2$.
 - (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x)$.
 - (b) Use the power rule to determine the derivative of $f(x)$.
3. Numerically determine the derivative of $\log(x)$ when $x = 3$. (Do not use any differentiation short cuts you might have learned in other classes.)
4. The number of crazed squirrels on campus grows at a rate of 10%. On Monday, October 2, there are 100 crazed squirrels on campus.
 - (a) Write an equation for $P(t)$, the number of crazed squirrels, t days after Monday, October 2.
 - (b) Sketch a graph of the number of crazed squirrels as a function of time. Be sure to label the axes and any intercept(s).
 - (c) On what day will there be approximately 1000 crazed squirrels on campus?
5.
 - (a) Sketch a function that has a positive first derivative for $x < 2$, a derivative of zero for $2 < x < 8$ and a negative first derivative for $x > 8$.
 - (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x - 2)$ and $f(x) - 2$. Make it clear which function is which.

6. Let $F(T)$ be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.
- Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)
 - What is the meaning of $F(4) = 3.20$?
 - What is the meaning of $F^{-1}(5) = -2.5$?
 - What are the units of $F'(T)$?
 - In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.
7. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.

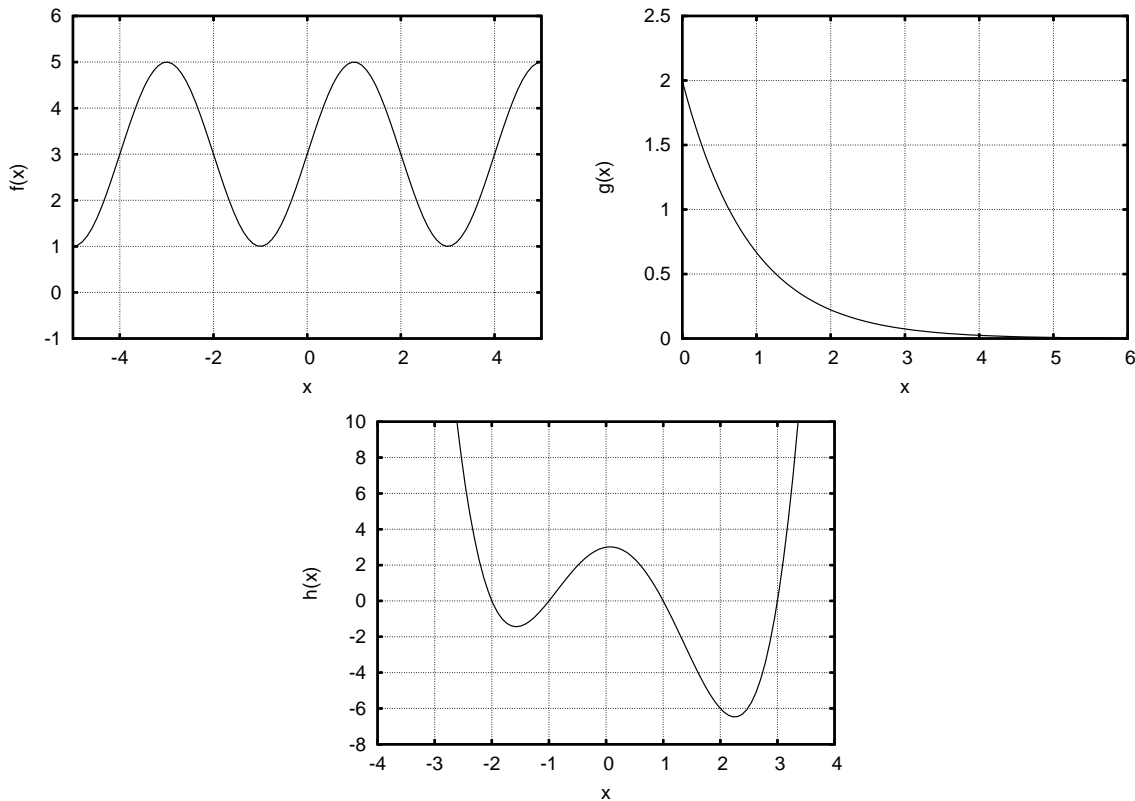


Figure 1: Graphs for problem 7.

8. Let $C(q)$ be the cost, in euros, of producing q pounds of Camembert cheese? Suppose that $C(10) = 25$ and that $C'(25) = 2.5$. Estimate $C(12)$.

9. For the function in Fig. 9:

- (a) Estimate $f'(1.5)$.
- (b) Estimate $f'(5)$.
- (c) Sketch $f'(x)$.

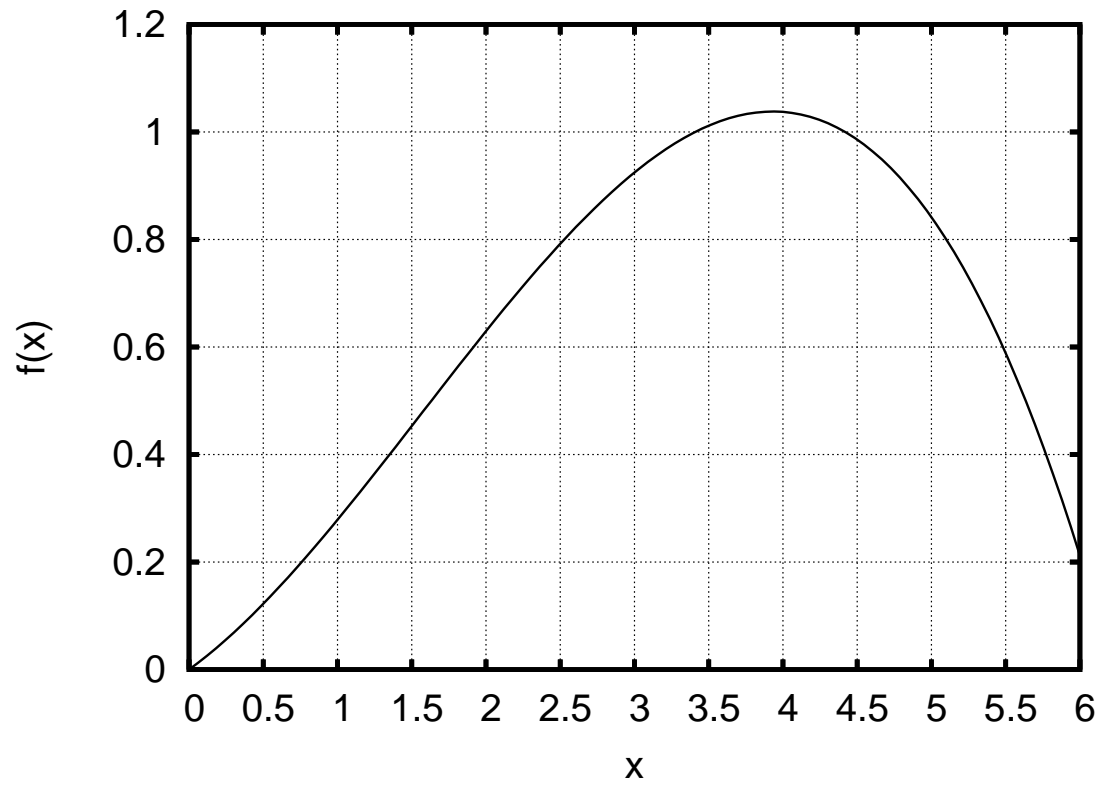


Figure 2: Graph for problem 9.

10. For the function in Fig. 3, which is larger? Briefly explain or illustrate your answer.

(a) $f(2)$ or $f(4)$?

(b) $f'(2)$ or $f'(4)$?

(c) $f'(2)$ or $\frac{f(4)-f(2)}{4-2}$?

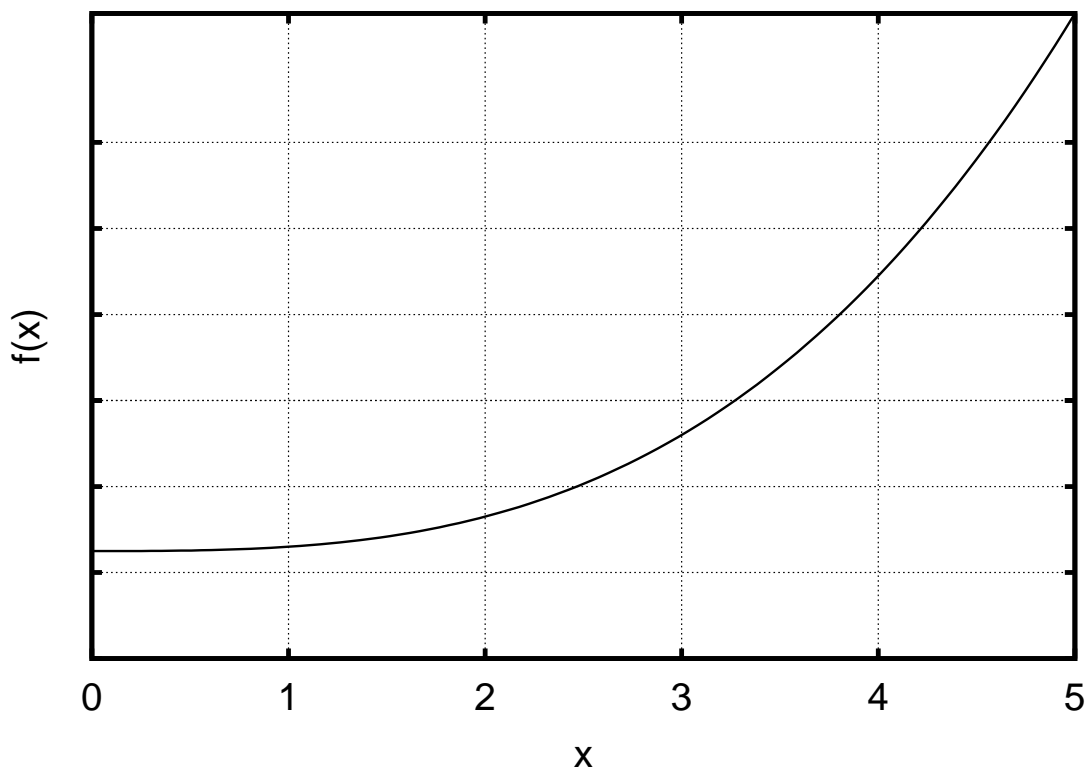


Figure 3: Graph for problem 10.