Optimizing

1. You recently acquired three alpacas and need to fence in a pasture so they don't wander off. Fortunately, you have a tall stone wall along one side of your property. So you'll need to build three walls, not four, to produce a nice rectangular field. You can afford 100 meters of fencing material. What dimensions should your field be so as to maximize the area available to the alpacas?

2. The strength of a rectangular beam of width w and height h is proportional to hw^2. A beam is to be cut from a log of radius r. What beam dimensions maximize the strength of this beam?

3. (From George W. Bluman, Problem Book for First Year Calculus, Springer-Verlag, 1984, p. 149.) An underdeveloped country, whose only export is coffee, can sell x tons of coffee per month on the international market at the price of $(300 - x/1000)$ dollars per ton. The cost of shipping x tons is 10 dollars per ton plus 1000 dollars overhead. What level of export will maximize the dollar income of the country?