

Calculus I Midterm

October 21–23, 2008

Important Directions

- **You may not collaborate on this exam; do not work with others.**
- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.
- This exam is untimed, but unless I hear to the contrary, I expect you to finish by noon on Thursday, October 23.
- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.
- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.
- To receive full credit on most of these problems you must show your work clearly.

1. Short Answers. No explanations needed.

- (a) Which function dominates as $x \rightarrow \infty$: x^3 or $900x$?
- (b) Which function dominates as $x \rightarrow \infty$: $x^{1/9}$ or x^{-9} ?
- (c) What is the period of $g(x) = 3 \cos(7x) + 4$?
- (d) Solve for t : $7^{t+1} = 711$.

2. Let $f(x) = 3x^2$.

- (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x)$.
- (b) Use the power rule to determine the derivative of $f(x)$.

3. Numerically determine the derivative of $\log(3x)$ when $x = 2$. (Do not use any differentiation short cuts you might have learned in other classes.)

4. Alan Greenspan's faith in the free market decreases by 9% every month after October 1. On October 1, Greenspan's faith was rated at 93. (In this scale, 100 corresponds to Milton Friedman, and 0 indicates a belief in rigid, Soviet-style central planning.)

- (a) Write an equation for $F(t)$, Alan Greenspan's faith t months after October 1.
- (b) Sketch a graph of Greenspan's faith as a function of time. Be sure to label the axes and any intercept(s).
- (c) When will Greenspan's faith be approximately 25?

5. (a) Sketch a function that has a positive first derivative for $x < -1$, a derivative of zero for $-1 < x < 5$ and a negative first derivative for $x > 5$.
- (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x - 3)$ and $f(x) - 3$. Make it clear which function is which.
- (c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 5a that is invertible? Why or why not?
6. Let $F(T)$ be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.
- (a) Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)
- (b) What is the meaning of $F(4) = 3.20$?
- (c) What is the meaning of $F^{-1}(5) = -2.5$?
- (d) What are the units of $F'(T)$?
- (e) In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.
7. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.

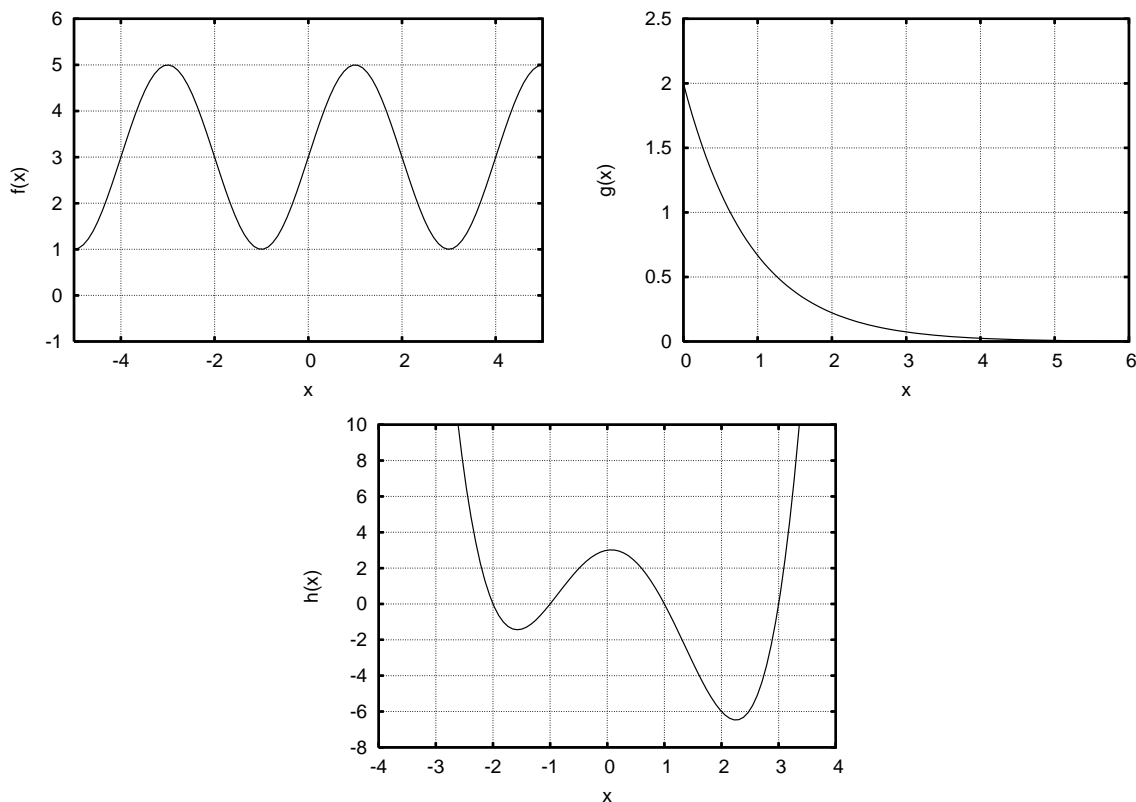


Figure 1: Graphs for problem 7.

8. Let $C(q)$ be the cost, in euros, of producing q pounds of Camembert cheese. Suppose that $C(10) = 25$ and that $C'(10) = 2.1$. Estimate $C(8.5)$.
9. For the function in Fig. 9:
- (a) Estimate $f'(1.5)$.
 - (b) Estimate $f'(5)$.
 - (c) Sketch $f'(x)$.

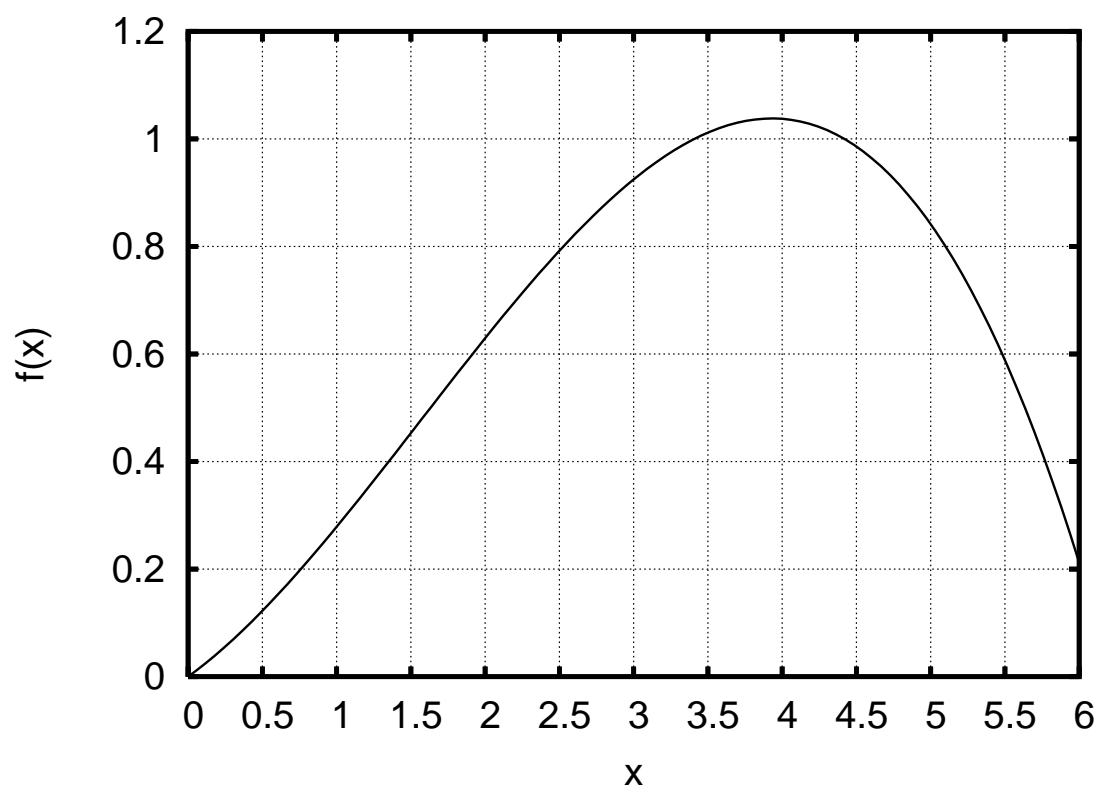


Figure 2: Graph for problem 9.

10. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate your answer.

- (a) Which is larger $f(2)$ or $f(4)$?
- (b) Which is larger $f'(2)$ or $f'(4)$?
- (c) Which is larger $f'(2)$ or $\frac{f(4)-f(2)}{4-2}$?
- (d) What is the sign of $f''(3)$?

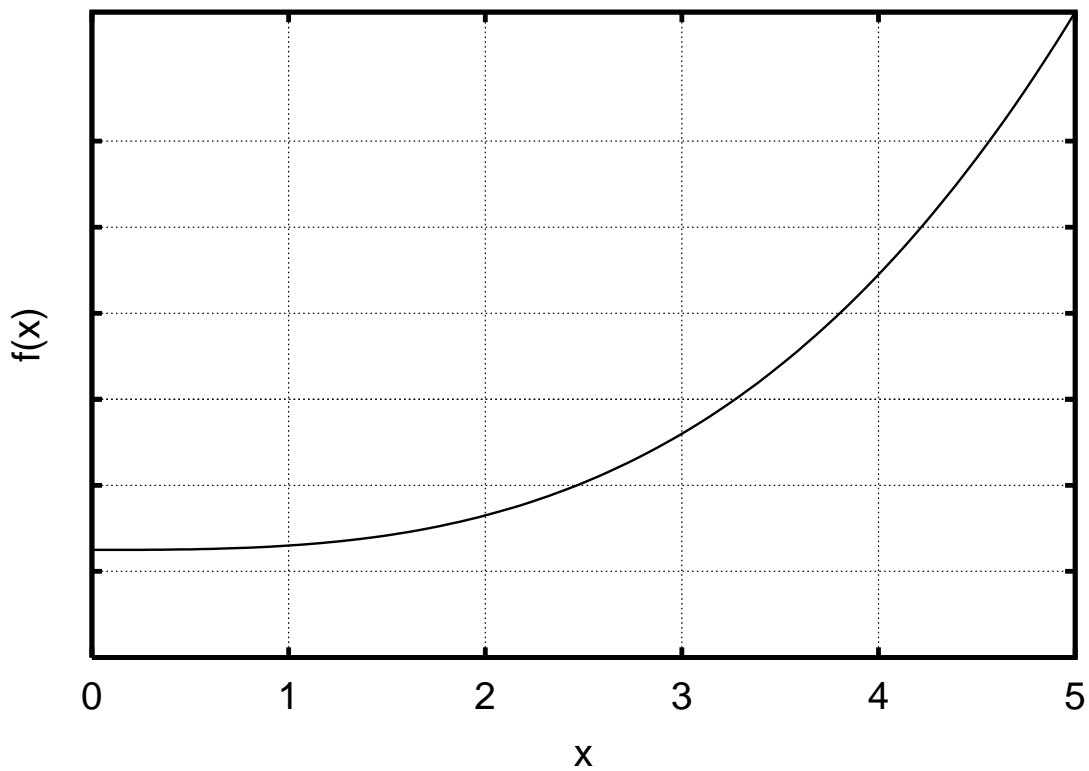


Figure 3: Graph for problem 10.

11. Answer the following questions for the function $h(x)$ shown in Fig. 4. Briefly explain or illustrate your answer.

- (a) For what values of x is $h(x)$ positive?
- (b) For what values of x is $h'(x)$ positive?
- (c) For what values of x is $h''(x)$ positive?

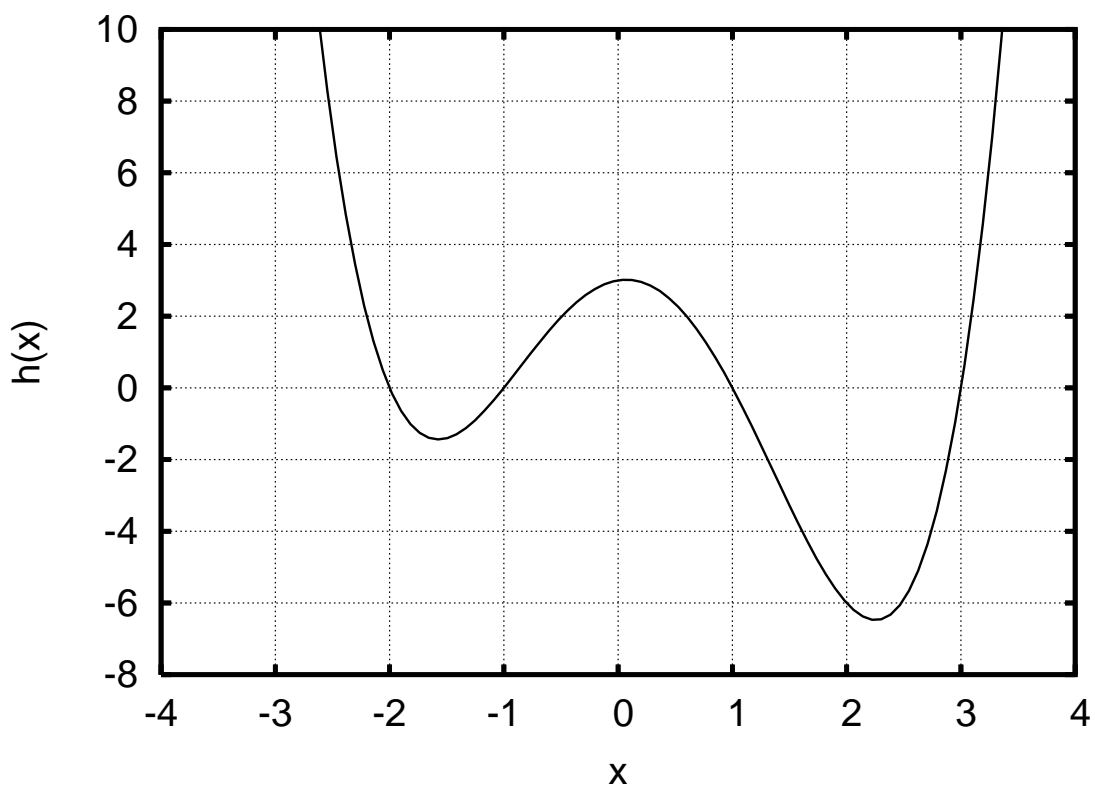


Figure 4: Graph for problem 11.