## Calculus I Midterm

October 10–12, 2006

## Important Directions

- You may not collaborate on this exam; do not work with others.
- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.
- This exam is untimed, but unless I hear to the contrary, I expect you to finish by 8:00am on Friday, October 13.
- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.
- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.
- To receive full credit on most of these problems you must show your work clearly.
- 1. Short Answers. No explanations needed.
  - (a) Which function dominates as  $x \to \infty$ :  $x^3$  or 900x?
  - (b) Which function dominates as  $x \to \infty$ :  $x^{1/9}$  or  $x^{-9}$ ?
  - (c) What is the period of  $g(x) = 3\cos(2x) + 4$ ?
- 2. Let  $f(x) = 6x^2$ .
  - (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of f(x).
  - (b) Use the power rule to determine the derivative of f(x).
- 3. Numerically determine the derivative of log(x) when x = 3. (Do not use any differentiation short cuts you might have learned in other classes.)
- 4. The number of crazed squirrels on campus grows at a rate of 10%. On Monday, October 2, there are 100 crazed squirrels on campus.
  - (a) Write an equation for P(t), the number of crazed squirrels, t days after Monday, October 2.
  - (b) Sketch a graph of the number of crazed squirrels as a function of time. Be sure to label the axes and any intercept(s).
  - (c) On what day will there be approximately 1000 crazed squirrels on campus?
- 5. (a) Sketch a function that has a positive first derivative for x < 2, a derivative of zero for 2 < x < 8 and a negative first derivative for x > 8.
  - (b) Call this function f(x). On the same axes as your original graph, sketch f(x-2) and f(x)-2. Make it clear which function is which.

- 6. Let F(T) be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.
  - (a) Make a rough sketch a possible graph of F(T). (There are many possible answers.)
  - (b) What is the meaning of F(4) = 3.20?
  - (c) What is the meaning of  $F^{-1}(5) = -2.5$ ?
  - (d) What are the units of F'(T)?
  - (e) In practical terms what does F'(-4) = -.17 mean? Be sure to explain why the minus sign is there.
- 7. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.

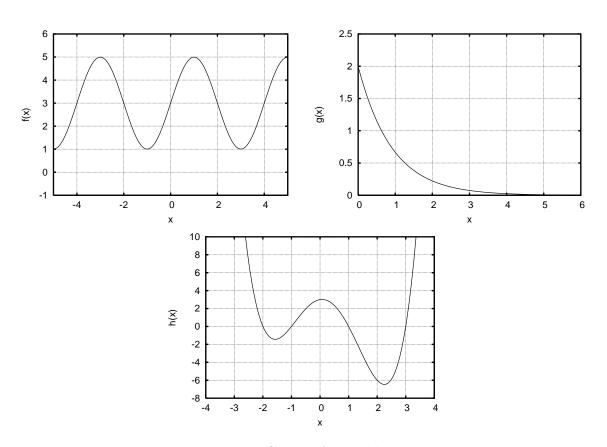


Figure 1: Graphs for problem 7.

8. Let C(q) be the cost, in euros, of producing q pounds of Camembert cheese? Suppose that C(10) = 25 and that C'(25) = 2.5. Estimate C(12).

- 9. For the function in Fig. 9:
  - (a) Estimate f'(1.5).
  - (b) Estimate f'(5).
  - (c) Sketch f'(x).

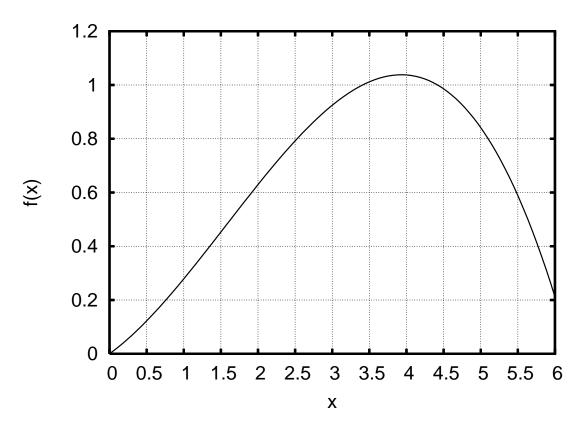


Figure 2: Graph for problem 9.

10. For the function in Fig. 3, which is larger? Briefly explain or illustrate you answer.

- (a) f(2) or f(4)?
- (b) f'(2) or f'(4)?
- (c) f'(2) or  $\frac{f(4)-f(2)}{4-2}$ ?

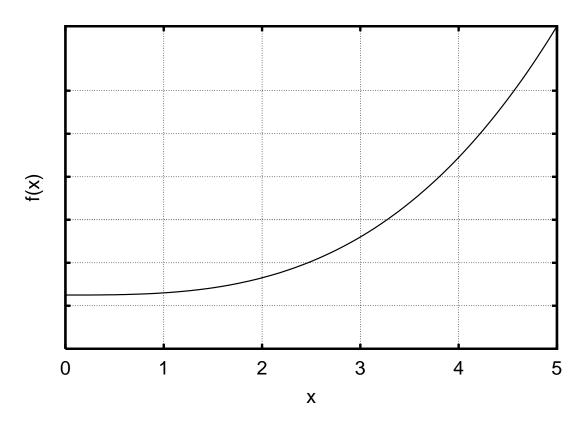


Figure 3: Graph for problem 10.