Calculus I Midterm

October 19–20, 2010

Important Directions

- **You may not collaborate on this exam; do not work with others.**

- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.

- This exam is untimed, but unless I hear to the contrary, I expect you to finish by 10am on Thursday, October 21.

- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.

- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.

- To receive full credit on most of these problems you must show your work clearly.

1. Short Answers. No explanations needed.

    (a) Which function dominates as $x \to \infty$: $99x^3$ or 3^x?

    (b) Which function dominates as $x \to \infty$: $x^{1/3}$ or x^{-8}?

    (c) What is the period of $g(x) = 4\cos(6x) + 3$?

    (d) Solve for t: $5^{t+2} = 423$.

2. Let $f(x) = 3x^2$.

    (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x)$.

    (b) Use the power rule to determine the derivative of $f(x)$.

3. Numerically determine the derivative of $\log(3x)$ when $x = 2$. (Do not use any differentiation short cuts you might have learned in other classes.)

4. A rabbit has been infected with worms. A drug is administered to the ailing rabbit that causes the worm infection load, measured in mg of worm mass, to decrease by 4% every hour. At 8am on Tuesday morning the rabbit's worm load is 75 mg.

    (a) Write an equation for $W(t)$, the rabbits' worm load t hours after 8am Tuesday.

    (b) Sketch a graph of $W(t)$. Be sure to label the axes and any intercept(s).

    (c) When will the rabbits worm load be approximately 25 mg?

5. (a) Sketch a function that has a positive first derivative for $x < -1$, a derivative of zero for $-1 < x < 5$ and a negative first derivative for $x > 5$.

   (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x-3)$ and $f(x) - 3$. Make it clear which function is which.

   (c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 5a that is invertible? Why or why not?

6. Let $F(T)$ be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.

   (a) Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)

   (b) What is the meaning of $F(4) = 3.20$?

   (c) What is the meaning of $F^{-1}(5) = -2.5$?

   (d) What are the units of $F'(T)$?

   (e) In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.

7. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.
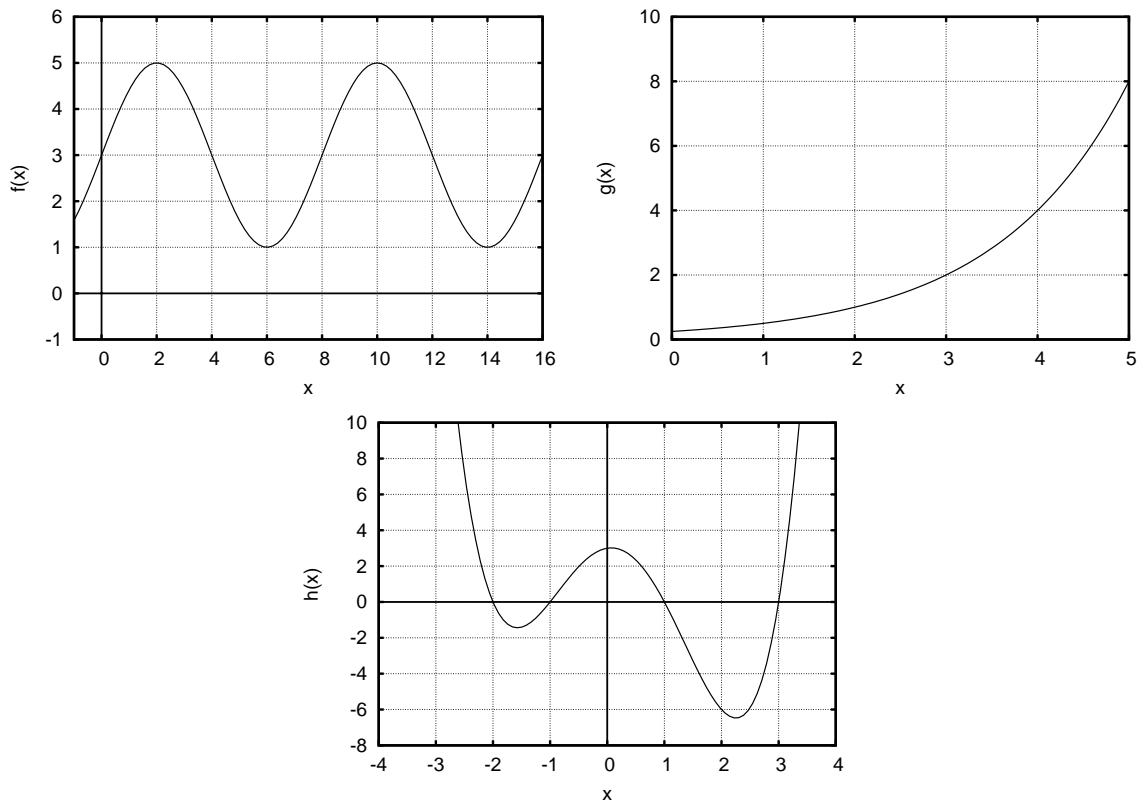

Figure 1: Graphs for problem 7.

8. Let $C(q)$ be the cost, in dollars, of q kilograms of carrots. Suppose that $C(20) = 25$ and that $C'(10) = 1.3$.

   (a) What are the units of $C'(q)$?

   (b) Estimate $C(18.3)$.

9. For the function in Fig. 9:

   (a) Estimate $f'(1.5)$.

   (b) Estimate $f'(5)$.
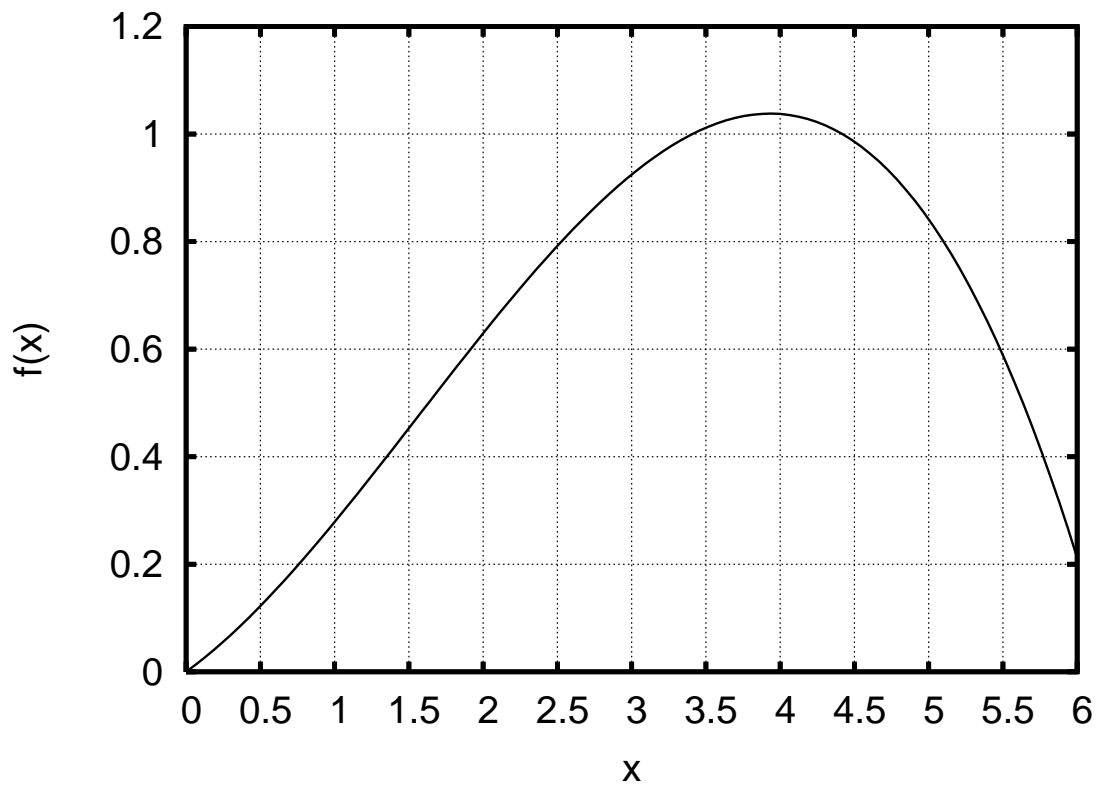
   (c) Sketch $f'(x)$.

Figure 2: Graph for problem 9.

10. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate you answer.

   (a) Which is larger $f(2)$ or $f(4)$?

   (b) Which is larger $f'(2)$ or $f'(4)$?

   (c) Which is larger $f'(2)$ or $\frac{f(4)-f(2)}{4-2}$?

   (d) What is the sign of $f''(3)$?

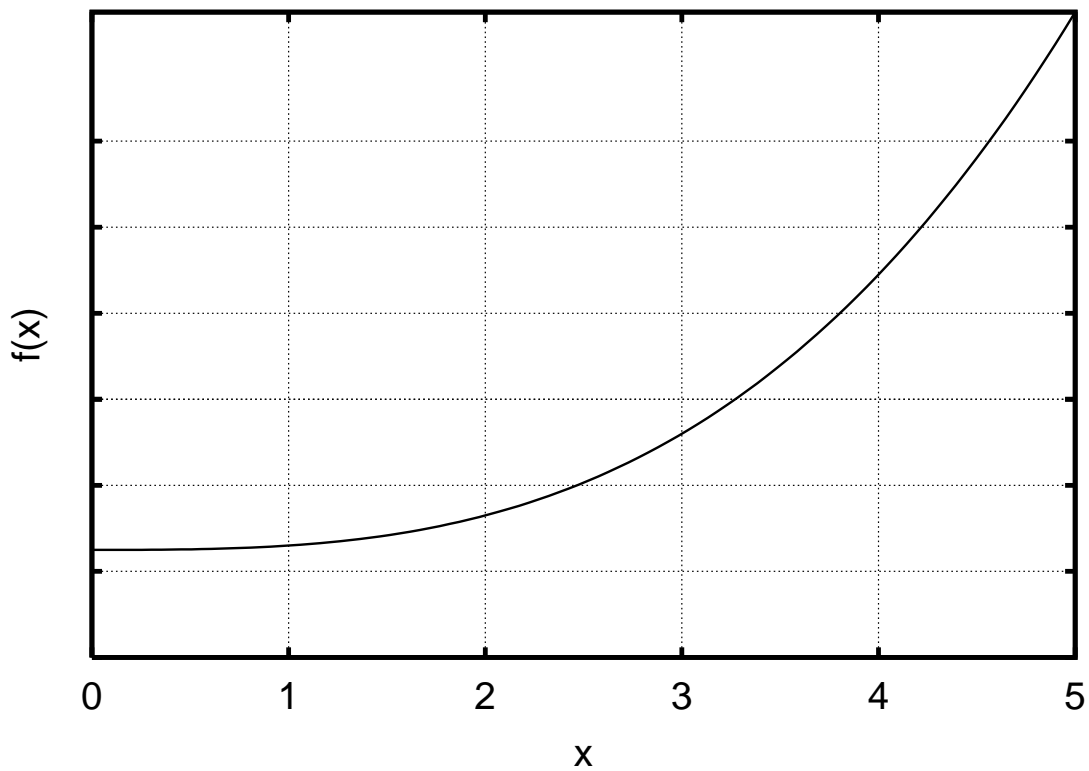
Figure 3: Graph for problem 10.

11. Answer the following questions for the function $h(x)$ shown in Fig. 4. Briefly explain or illustrate you answer.

    (a) For what values of x is $h(x)$ positive?

    (b) For what values of x is $h'(x)$ positive?
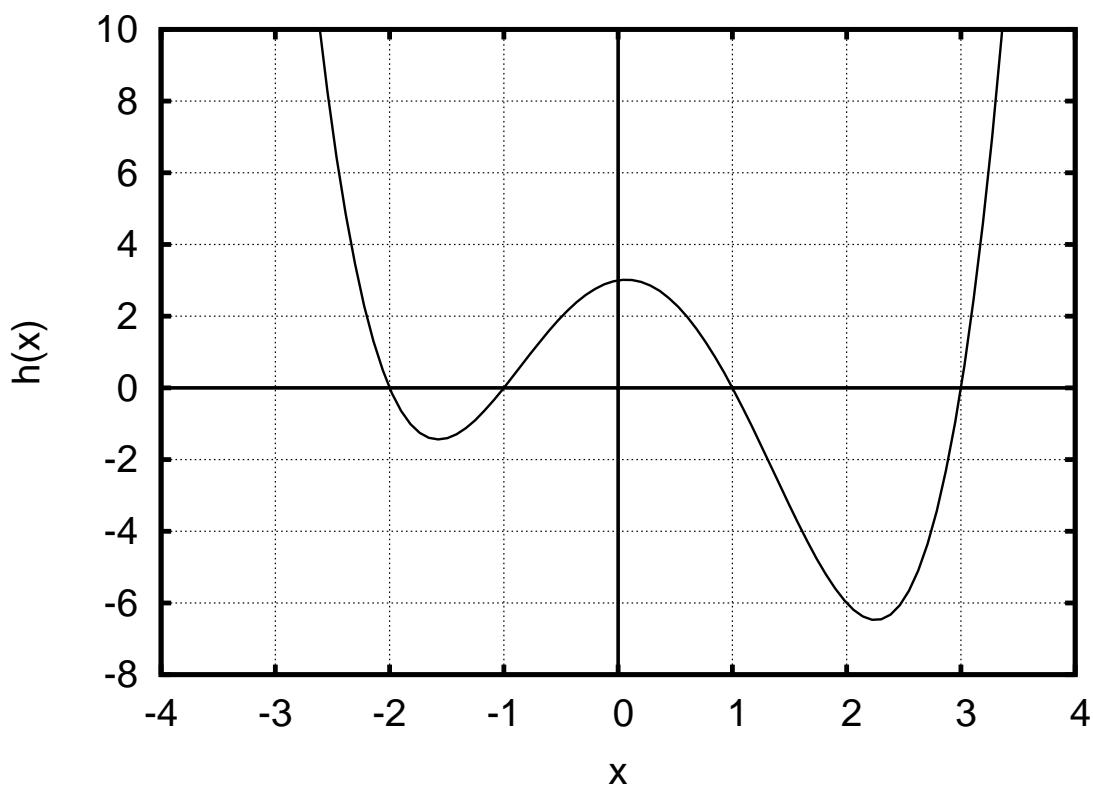
    (c) For what values of x is $h''(x)$ positive?

Figure 4: Graph for problem 11.