

# Calculus I Midterm

October 21–23, 2008

## Important Directions

- **You may not collaborate on this exam; do not work with others.**
- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.
- This exam is untimed, but unless I hear to the contrary, I expect you to finish by noon on Thursday, October 23.
- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.
- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.
- To receive full credit on most of these problems you must show your work clearly.

1. Short Answers. No explanations needed.

- (a) Which function dominates as  $x \rightarrow \infty$ :  $x^3$  or  $900x$ ?
- (b) Which function dominates as  $x \rightarrow \infty$ :  $x^{1/9}$  or  $x^{-9}$ ?
- (c) What is the period of  $g(x) = 3 \cos(7x) + 4$ ?
- (d) Solve for  $t$ :  $7^{t+1} = 711$ .

2. Let  $f(x) = 3x^2$ .

- (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of  $f(x)$ .
- (b) Use the power rule to determine the derivative of  $f(x)$ .

3. Numerically determine the derivative of  $\log(3x)$  when  $x = 2$ . (Do not use any differentiation short cuts you might have learned in other classes.)

4. Alan Greenspan's faith in the free market decreases by 9% every month after October 1. On October 1, Greenspan's faith was rated at 93. (In this scale, 100 corresponds to Milton Friedman, and 0 indicates a belief in rigid, Soviet-style central planning.)

- (a) Write an equation for  $F(t)$ , Alan Greenspan's faith  $t$  months after October 1.
- (b) Sketch a graph of Greenspan's faith as a function of time. Be sure to label the axes and any intercept(s).
- (c) When will Greenspan's faith be approximately 25?

5. (a) Sketch a function that has a positive first derivative for  $x < -1$ , a derivative of zero for  $-1 < x < 5$  and a negative first derivative for  $x > 5$ .
- (b) Call this function  $f(x)$ . On the same axes as your original graph, sketch  $f(x - 3)$  and  $f(x) - 3$ . Make it clear which function is which.
- (c) Is your  $f(x)$  invertible? Is it possible to come up with an  $f(x)$  that satisfies the criteria of question 5a that is invertible? Why or why not?
6. Let  $F(T)$  be the cost of heating your house, in dollars per day, when the average outside temperature is  $T$  Celsius degrees.
- (a) Make a rough sketch a possible graph of  $F(T)$ . (There are many possible answers.)
- (b) What is the meaning of  $F(4) = 3.20$ ?
- (c) What is the meaning of  $F^{-1}(5) = -2.5$ ?
- (d) What are the units of  $F'(T)$ ?
- (e) In practical terms what does  $F'(-4) = -.17$  mean? Be sure to explain why the minus sign is there.
7. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.

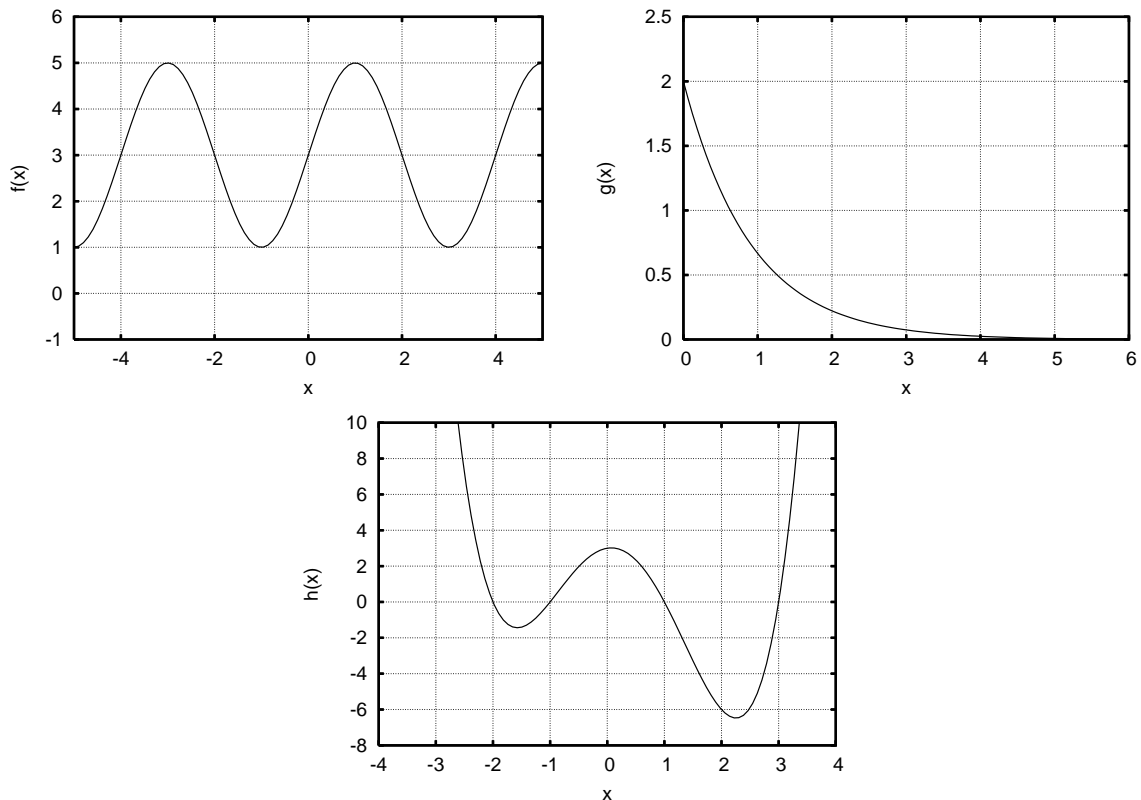


Figure 1: Graphs for problem 7.

8. Let  $C(q)$  be the cost, in euros, of producing  $q$  pounds of Camembert cheese. Suppose that  $C(10) = 25$  and that  $C'(25) = 2.1$ . Estimate  $C(8.5)$ .
9. For the function in Fig. 9:
- (a) Estimate  $f'(1.5)$ .
  - (b) Estimate  $f'(5)$ .
  - (c) Sketch  $f'(x)$ .

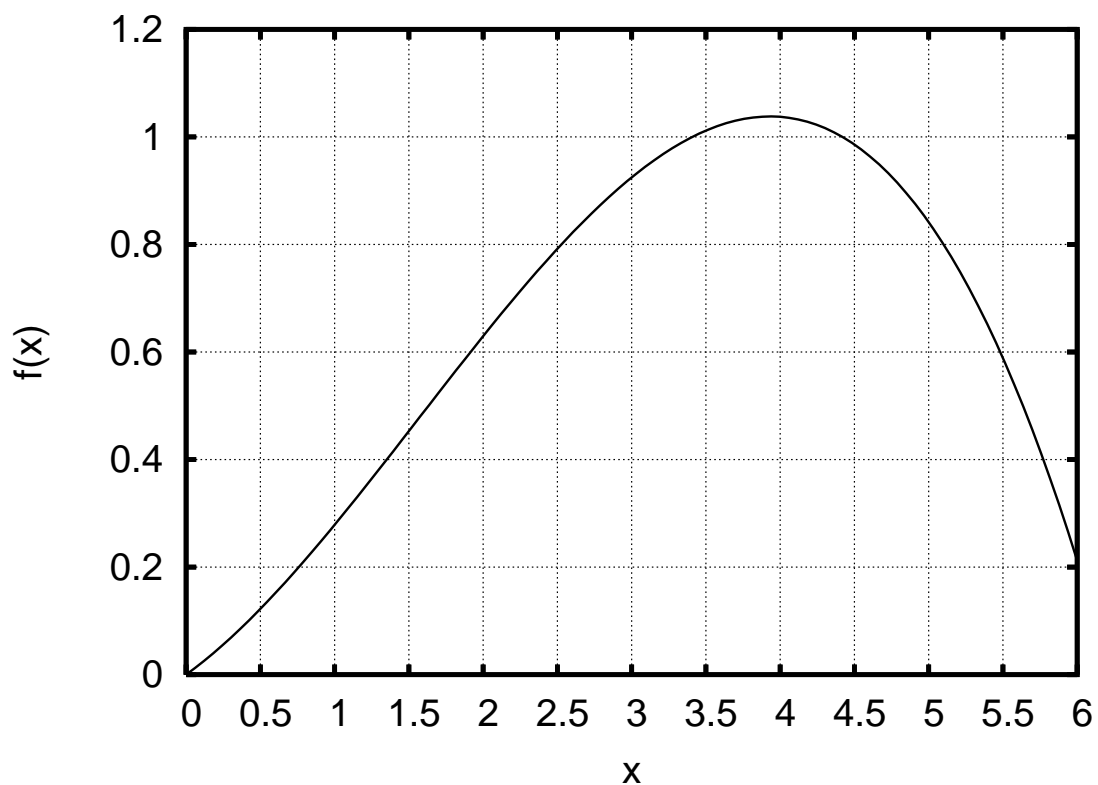


Figure 2: Graph for problem 9.

10. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate your answer.

- (a) Which is larger  $f(2)$  or  $f(4)$ ?
- (b) Which is larger  $f'(2)$  or  $f'(4)$ ?
- (c) Which is larger  $f'(2)$  or  $\frac{f(4)-f(2)}{4-2}$ ?
- (d) What is the sign of  $f''(3)$ ?

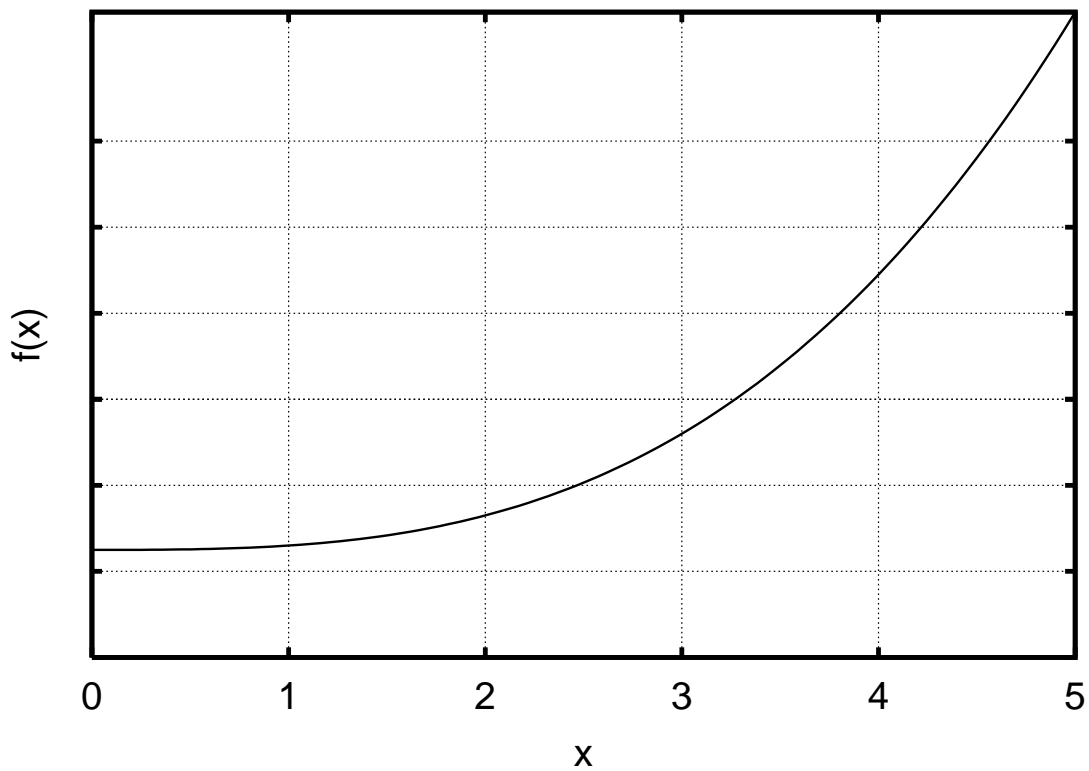


Figure 3: Graph for problem 10.

11. Answer the following questions for the function  $h(x)$  shown in Fig. 4. Briefly explain or illustrate your answer.

- (a) For what values of  $x$  is  $h(x)$  positive?
- (b) For what values of  $x$  is  $h'(x)$  positive?
- (c) For what values of  $x$  is  $h''(x)$  positive?

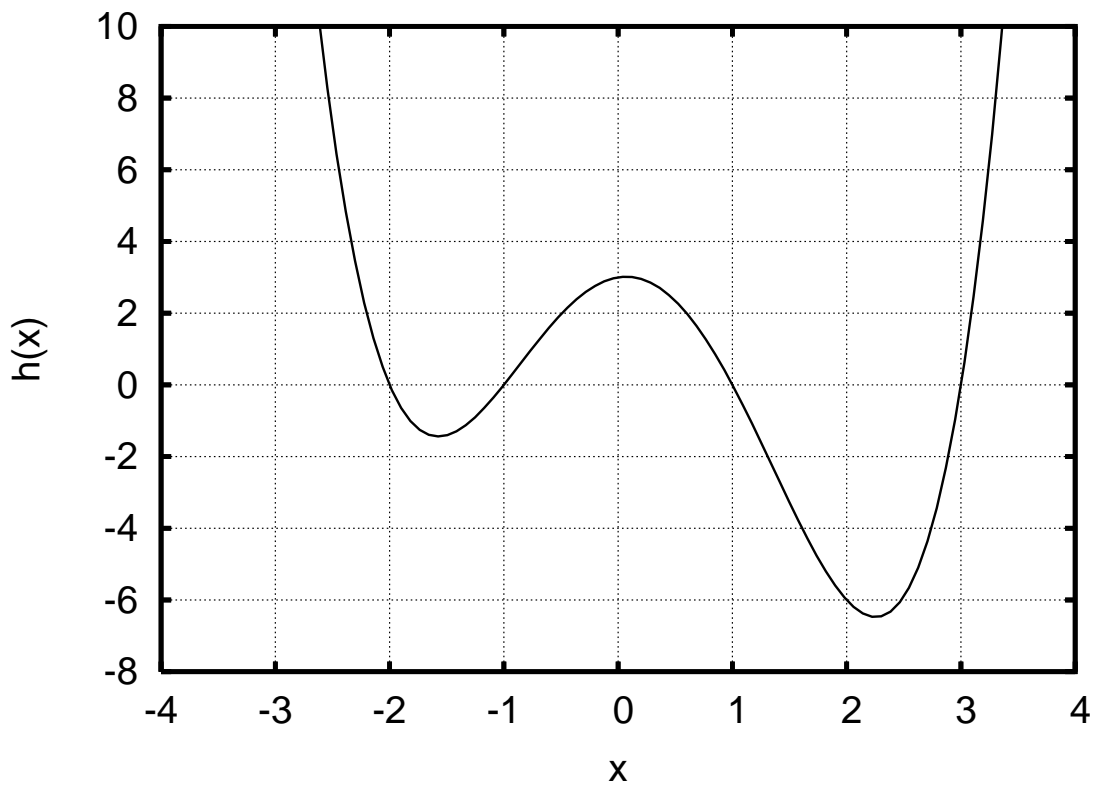


Figure 4: Graph for problem 11.