Calculus Lab: Analyzing Functions

30 October 2012

Hand this in as part of the homework assignment due this Friday, November 2.

Consider the function $f(x) = x^3 - ax + 2$, where a is a constant. We will vary this constant and see how the function changes. The constant a can be both positive and negative.

1. Produce several plots of $f(x)$ for different values of a that illustrate the effect that a has on the function. Choose your x-range carefully so that you can see all the interesting behavior of the function. You will probably want to use wolfram alpha to make plots, but for your write-up you don't need printouts; just make sketches of the function.
2. Use algebra to the critical points. Your answer will depend on a.
   (a) For what values of a are there two critical points?
   (b) For what values of a are there one critical point?
   (c) For what values of a are there no critical points?
3. For what value of a does $f(x)$ have a local maximum at $y = 6$?
4. Find any inflection points of $f(x)$.

(Optional: Don't worry if you don't have time for this.) Consider the function $g(x) = ax + \frac{b}{x}$ where a and b are parameters and x is positive.

1. What is the shape of this function? Why? (It may help to plot ax and $\frac{b}{x}$ together on the same axes.)
2. Find the coordinates for any critical points.
3. If you increase the value of a, what happens to the critical point? Why?
4. If you increase the value of b, what happens to the critical point? Why?