Chapter 2.5: The Second Derivative
Calculus I
College of the Atlantic. Fall 2014

1. Laura says:

   > I feel bad today, but I'm feeling better than yesterday, and I seem to be improving faster and faster.

   Let $f(t)$ be Laura's health as a function of time. Based on her statement, what can you say about the signs of $f(t)$, $f'(t)$, and $f''(t)$?

2. Representative Michaud says:

   > The defense budget will increase this year, but not by as much as it increased last year.

   Let $B(t)$ be the defense budget as a function of time. Based on Congressman Michaud's remarks, what can you say about the signs of $B'(t)$ and $B''(t)$?

3. Let $f(t)$ be the number of inches of rain that has fallen since midnight, where t is the time in hours. Interpret the following in practical terms, giving units.

   (a) $f(10) = 1.4$
   (b) $f'(1) = 0.1$
   (c) $f''(10) = -0.2$
   (d) $f^{-1}(1) = 3$
   (e) $(f^{-1})'(1.4) = 3$

4. Sketch a graph of a function that has the following properties: Its second derivative is positive everywhere and the first derivative is negative if $x < -2$ and positive if $x > -2$.

5. Sketch a graph of a function that has the following properties: Its second derivative is positive everywhere and the first derivative is positive if $x < -2$ and negative if $x > -2$.

6. The plot below is of a $f'(x)$. For what values of x is:

   (a) $f(x)$ increasing?

   (b) $f(x)$ decreasing?

   (c) $f'(x)$ positive?

   (d) $f'(x)$ negative?

   (e) $f''(x)$ positive?

   (f) $f''(x)$ negative?

7. Sketch $f''(x)$, given the $f'(x)$ in Fig. 1.
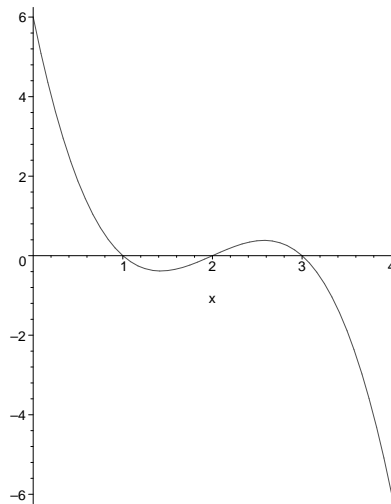
8. Sketch a possible $f(x)$ that corresponds to the $f'(x)$ in Fig. 1.

Figure 1: A plot of $f'(x)$, not $f(x)$.