Calculus I Midterm

October 18–20, 2016

Important Directions

- **You may not collaborate on this exam; do not work with others.**

- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.

- This exam is untimed, but unless I hear to the contrary, I expect you to finish by 9am on Wednesday, October 22.

- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.

- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.

- To receive full credit on most of these problems you must show your work clearly.

1. What is the period of $g(x) = 5\sin(3\pi x) - 3$?

2. Solve for t: $4^{t+2} = 200$.

3. Let $f(x) = 3x^2$.

   (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x)$.

   (b) Use the power rule to determine the derivative of $f(x)$.

4. Numerically determine the derivative of $4\sin(2x)$ when $x = 3$. Use radians. (To evaluate the derivative, use difference quotients. Do not use any differentiation short cuts you might have learned in other classes.)

5. A rabbit has been infected with worms. A drug is administered to the ailing rabbit that causes the worm infection load, measured in mg of worm mass, to decrease by 4% every hour. At 8am on Tuesday morning the rabbit's worm load is 75 mg.

   (a) Write an equation for $W(t)$, the rabbits' worm load t hours after 8am Tuesday.

   (b) Sketch a graph of $W(t)$. Be sure to label the axes and any intercept(s).

   (c) When will the rabbit's worm load be approximately 25 mg?

6. (a) Sketch a function that has a negative first derivative for $x < 2$ and a positive first derivative for $x > 2$.

    (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x-3)$ and $f(x) - 3$. Make it clear which function is which.

    (c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 6a that is invertible? Why or why not?

7. The dose D of a painkiller depends on the weight W of a patient. Thus, $D = f(W)$, where D is in milligrams and W is in pounds. Explain the practical meaning of the following statements. Be sure to include units with your answers:

    (a) $f(140) = 120$

    (b) $f'(140) = 3$

    (c) $f^{-1}(50) = 100$

8. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.
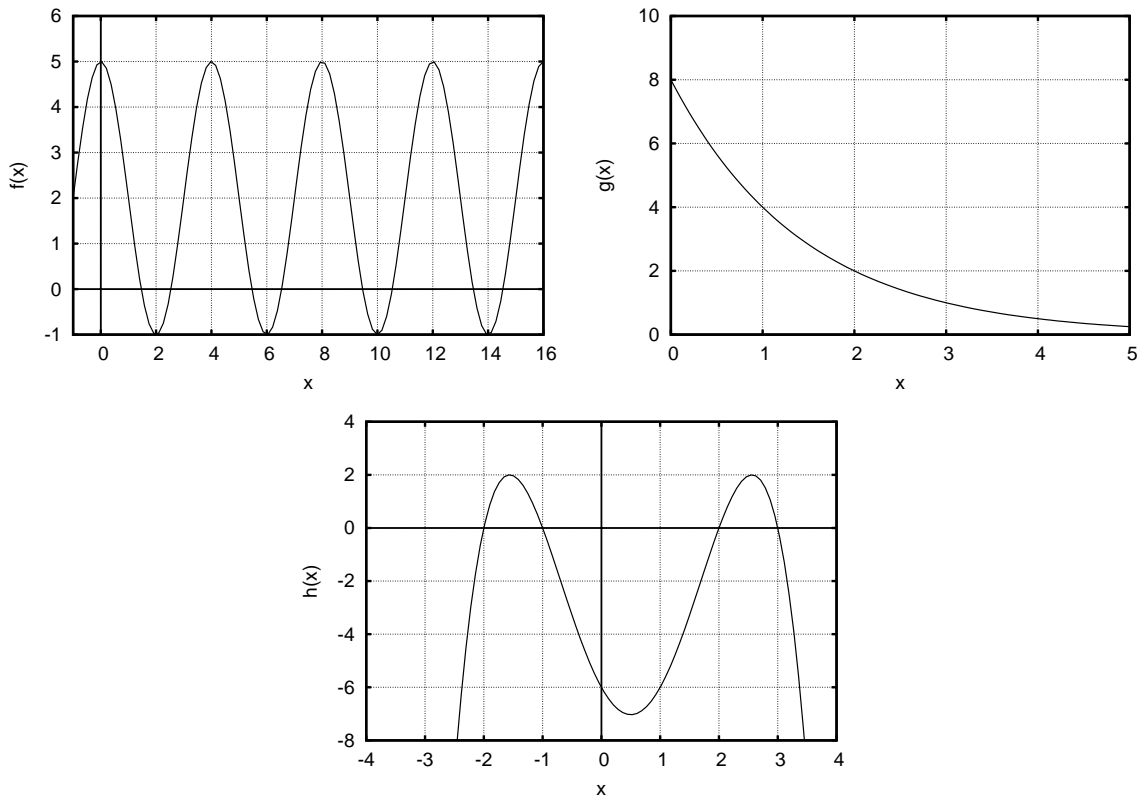

Figure 1: Graphs for problem 8.

9. Let $C(q)$ be the cost, in euros, of q kilograms of organic tofu. Suppose that $C(20) = 50$ and that $C'(20) = 1.3$.

    (a) What are the units of $C'(q)$?

    (b) Estimate $C(18)$.

10. For the function in Fig. 10:

    (a) Estimate $f'(1.5)$.

    (b) Estimate $f'(5)$.

    (c) Sketch $f'(x)$.

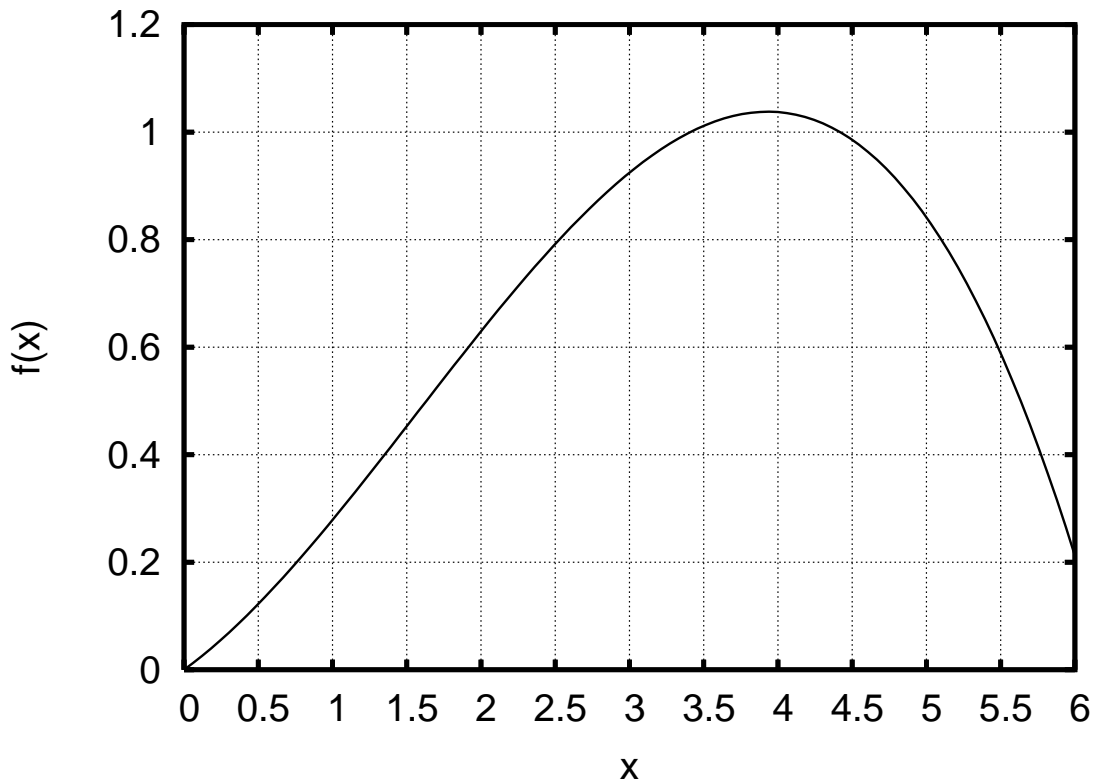For parts (a) and (b), be sure to show your work.

Figure 2: Graph for problem 10.

11. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate you answer.

(a) Which is larger $f(2)$ or $f(4)$?

(b) Which is larger $f(4.5)$ or $f(5.5)$?

(c) Which is larger $f'(2)$ or $f'(4)$?

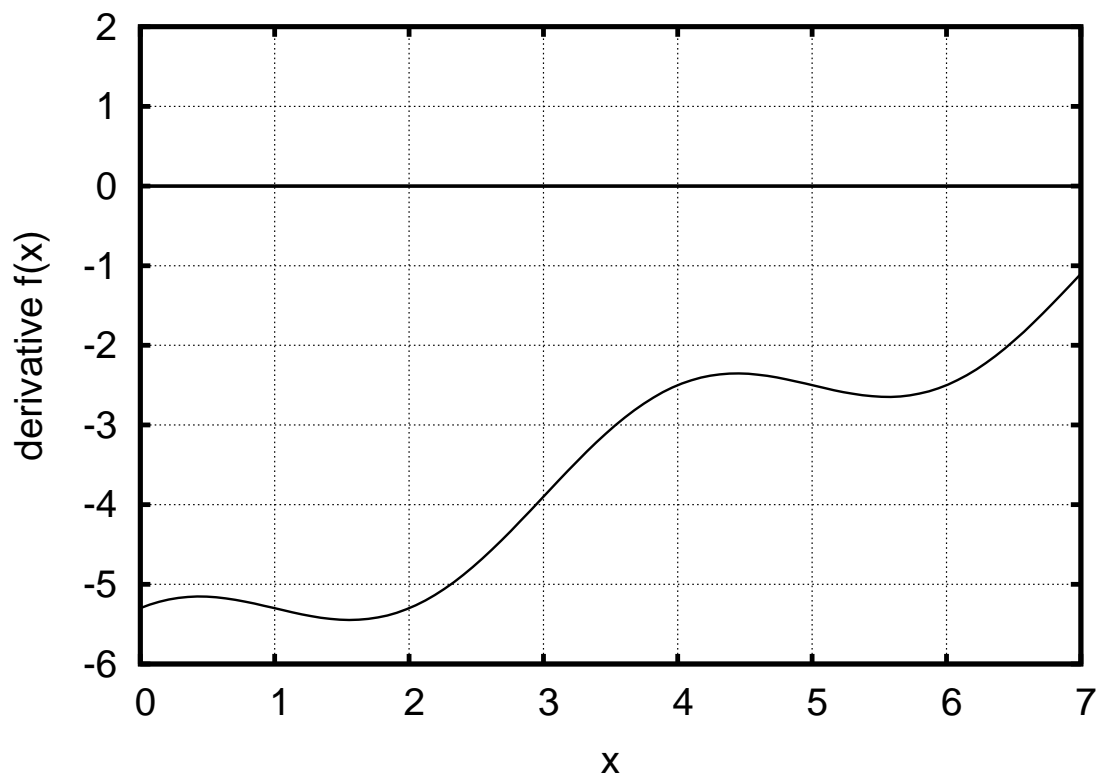(d) Which is larger $f''(2)$ or $f''(3)$?

Figure 3: Graph for problem 11. Note that this is a plot of $f'(x)$, not $f(x)$.

12. (a) Make a qualitatively accurate sketches of $h'(x)$ and $h''(x)$ for the function $h(x)$ shown in Fig. 4. Please make the sketches on separate axes, one above the other.
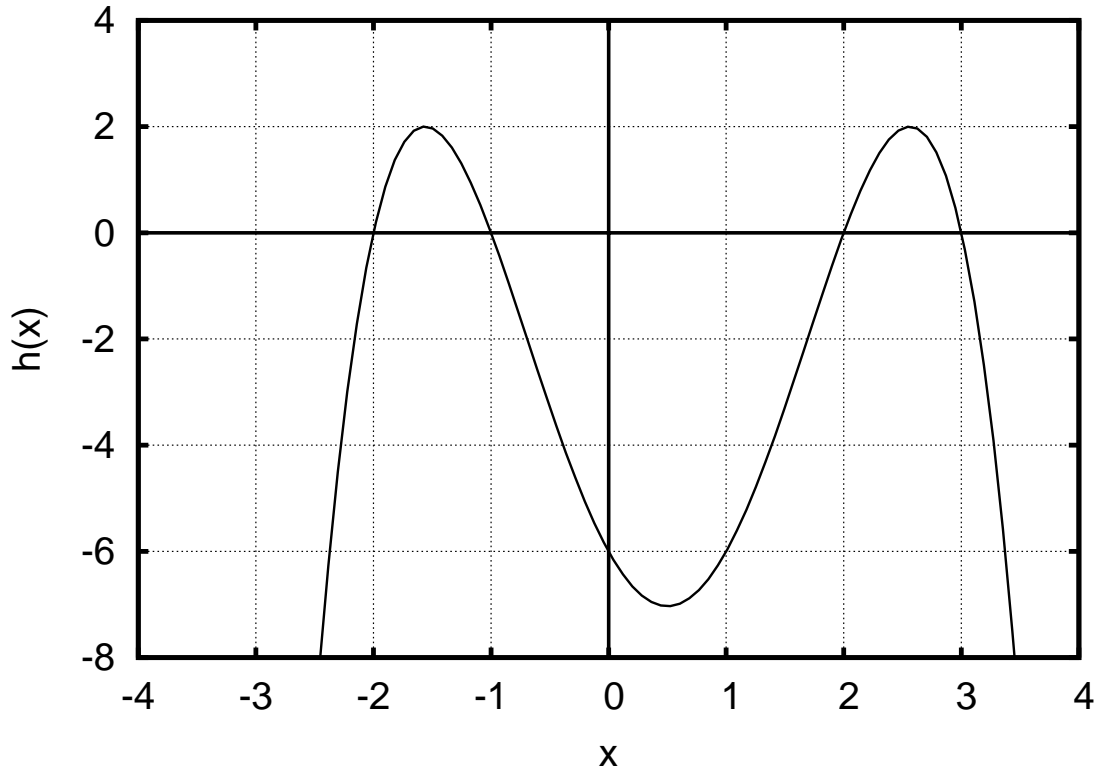
(b) For approximately what range(s) of x values is $h(x)$ concave down?

Figure 4: Graph for problem 12.