

Calculus I Midterm

October 16-17, 2018

Important Directions

- **You may not collaborate on this exam; do not work with others.**
- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.
- This exam is untimed, but unless I hear to the contrary, I expect you to finish by 6pm on Wednesday, October 17.
- When you are done with the exam, please slip it under my office door. Don't put it my mailbox.
- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.
- To receive full credit on most of these problems you must show your work clearly.

1. What is the period of $g(x) = -3\sin(2\pi x) + 5$?
2. Solve for t : $6^{t-1} = 300$.
3. Let $f(x) = 3x^2 + x$.
 - (a) Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x)$.
 - (b) Use the power rule to determine the derivative of $f(x)$.
4. Numerically determine the derivative of $3\log(5x)$ when $x = 2$. (To evaluate the derivative, use difference quotients. Do not use any differentiation short cuts you might have learned in other classes.)
5. A rabbit has been infected with worms. A drug is administered to the ailing rabbit that causes the worm infection load, measured in mg of worm mass, to decrease by 4% every hour. At 8am on Tuesday morning the rabbit's worm load is 75 mg.
 - (a) Write an equation for $W(t)$, the rabbits' worm load t hours after 8am Tuesday.
 - (b) Sketch a graph of $W(t)$. Be sure to label the axes and any intercept(s).
 - (c) When will the rabbit's worm load be approximately 25 mg?

6. (a) Sketch a function that has a positive first derivative for $x < 0$ and a negative first derivative for $x > 0$.
- (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x - 3)$ and $f(x) - 3$. Make it clear which function is which.
- (c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 6a that is invertible? Why or why not?
7. Let $F(T)$ be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.
- (a) Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)
- (b) What is the meaning of $F(4) = 3.20$?
- (c) What is the meaning of $F^{-1}(5) = -2.5$?
- (d) What are the units of $F'(T)$?
- (e) In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.
8. For each of the graphs in Fig. 1, find a possible formula for the function. If there are any constants in your formula, indicate if those constants are positive or negative.

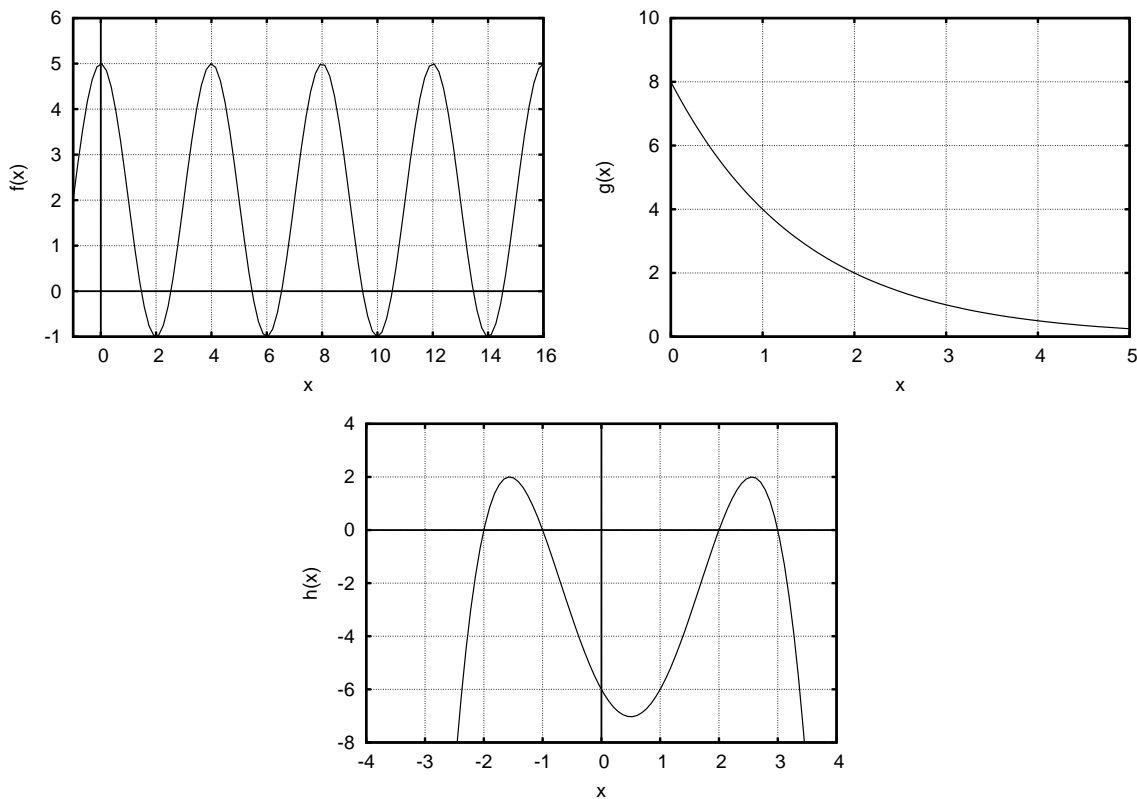


Figure 1: Graphs for problem 8.

9. Let $C(q)$ be the cost, in euros, of q kilograms of organic tofu. Suppose that $C(20) = 50$ and that $C'(20) = 1.3$.

(a) What are the units of $C'(q)$?

(b) Estimate $C(18)$.

10. For the function in Fig. 10:

(a) Estimate $f'(1.5)$.

(b) Sketch $f'(x)$.

For part (a) be sure to show your work.

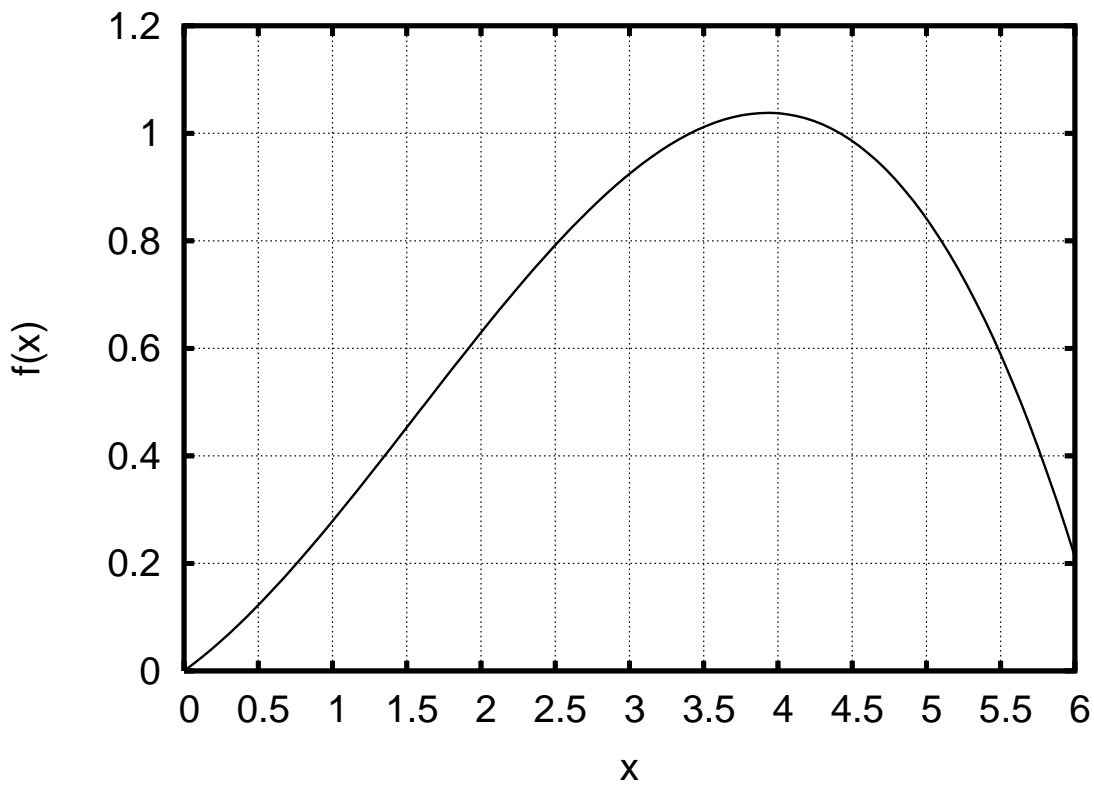


Figure 2: Graph for problem 10.

11. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate your answer.

- (a) Which is larger $f(2)$ or $f(4)$?
- (b) Which is larger $f(4.5)$ or $f(5.5)$?
- (c) Which is larger $f'(2)$ or $f'(4)$?
- (d) Which is larger $f''(2)$ or $f''(3)$?

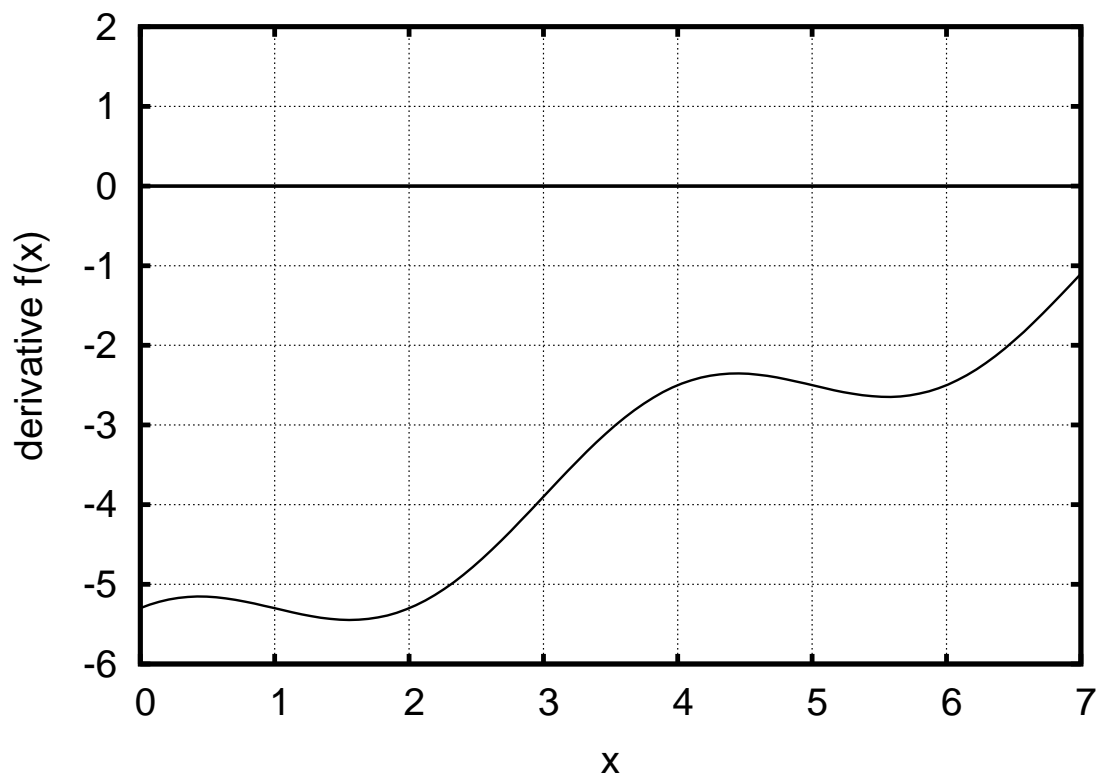


Figure 3: Graph for problem 11. Note that this is a plot of $f'(x)$, not $f(x)$.

12. (a) Make a qualitatively accurate sketches of $h'(x)$ and $h''(x)$ for the function $h(x)$ shown in Fig. 4. Please make the sketches on separate axes, one above the other.
- (b) For approximately what range(s) of x values is $h(x)$ concave down?

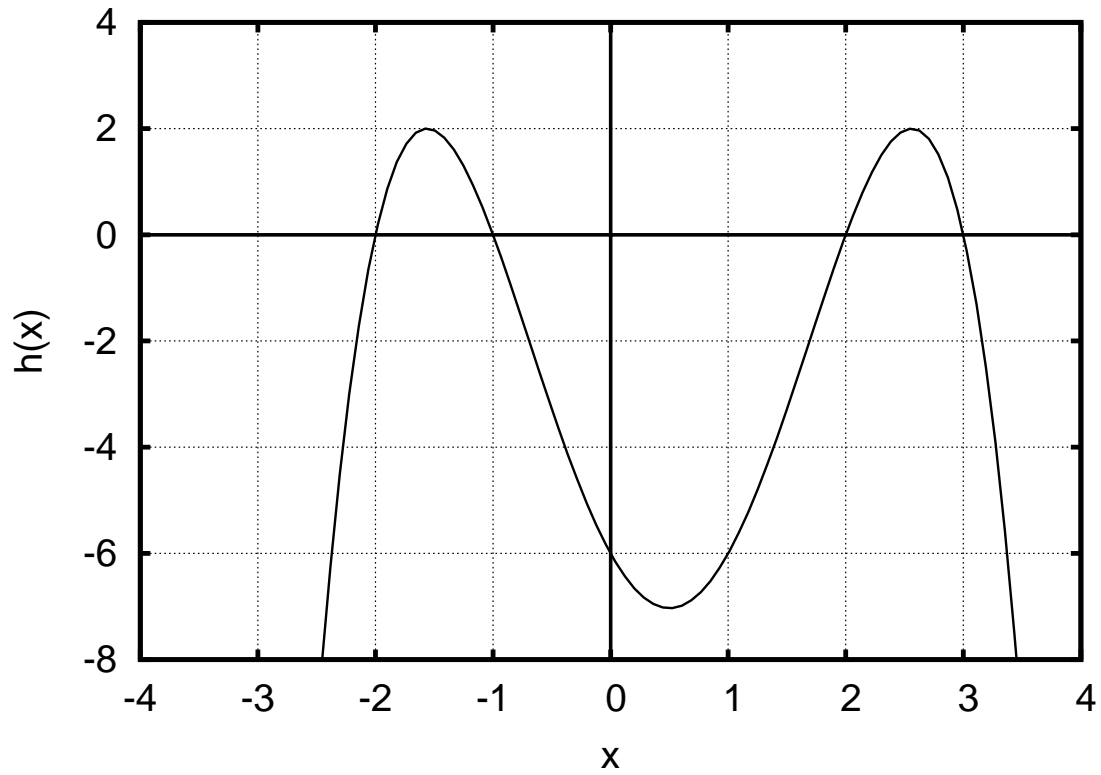


Figure 4: Graph for problem 12.