Chapter 2.5: The Second Derivative
Calculus I
College of the Atlantic. October 10, 2022

1. Laura says:

   > I feel bad today, but I'm feeling better than yesterday, and I seem to be improving faster and faster.

   Let $f(t)$ be Laura's health as a function of time. Based on her statement, what can you say about the signs of $f(t)$, $f'(t)$, and $f''(t)$?

2. Representative Michaud says:

   > The defense budget will increase this year, but not by as much as it increased last year.

   Let $B(t)$ be the defense budget as a function of time. Based on Congressman Michaud's remarks, what can you say about the signs of $B'(t)$ and $B''(t)$?

3. Let $f(t)$ be the number of inches of rain that has fallen since midnight, where t is the time in hours. Interpret the following in practical terms, giving units.

   (a) $f(10) = 1.4$

   (b) $f'(1) = 0.1$

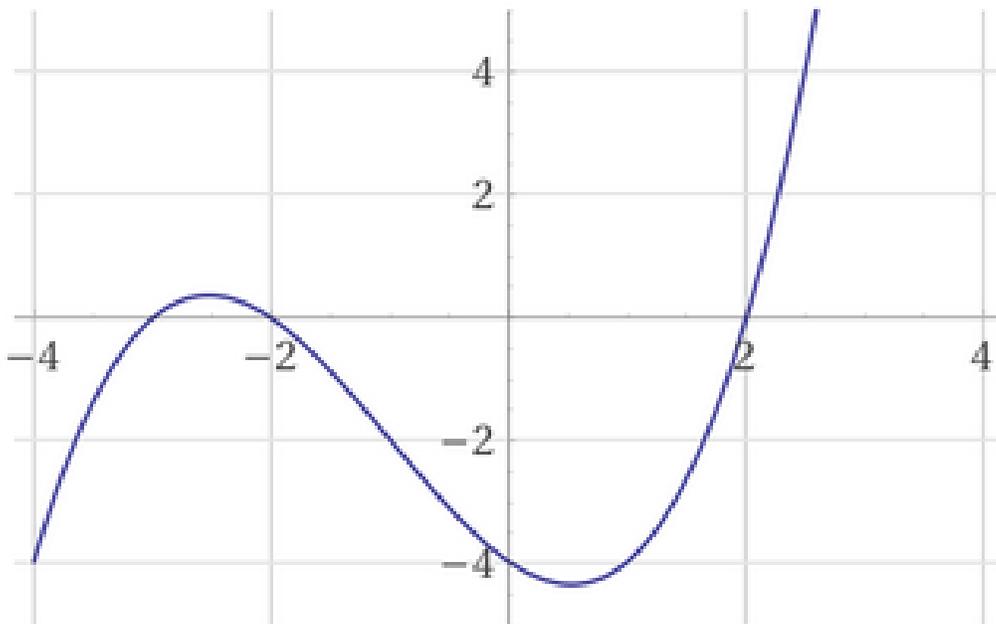   (c) $f''(10) = -0.2$

   (d) $f^{-1}(1) = 3$

   (e) $(f^{-1})'(1.4) = 3$

4. A function (not its derivative) is plotted in Fig. 4.

    (a) For what values of x is $f(x)$ positive?

    (b) For what values of x is $f(x)$ negative?

    (c) For what values of x is $f'(x)$ positive?

    (d) For what values of x is $f'(x)$ negative?

    (e) For what values of x is $f''(x)$ positive?

    (f) For what values of x is $f''(x)$ negative?

5. The plot below is of $f'(x)$. For what values of x is:

    (a) $f(x)$ increasing?

    (b) $f(x)$ decreasing?

    (c) $f'(x)$ positive?

    (d) $f'(x)$ negative?

    (e) $f''(x)$ positive?

    (f) $f''(x)$ negative?

6. Sketch $f''(x)$, given the $f'(x)$ in Fig. 8.

7. Sketch a possible $f(x)$ that corresponds to the $f'(x)$ in Fig. 8.

8. Sketch another possible $f(x)$ that corresponds to the $f'(x)$ in Fig. 8.