Lab 05

Calculus I

10 October 2022, College of the Atlantic

- Please work in groups of two or three
- Please write your answers on this sheet, make a scan of it as a pdf, and upload it google classroom at the end of lab. This assignment is not graded.

Names: _____

Part I: Limits

In this exercise you'll explore the idea of a limit outside of the context of derivatives.

1. First, we'll explore
$$\lim_{x\to 0} \frac{\sin(x)}{x} \,. \tag{1}$$
   (a) Evaluate this limit by letting x get closer and closer to zero. What is $\frac{\sin(x)}{x}$ if:
      i. $x = 0.1$
      ii. $x = 0.01$
      iii. $x = 0.001$

   (b) Make a conjecture for the value of the limit.

   (c) BTW, what would happen if you plugged in $x = 0$?

2. Next, we'll explore
$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) \,. \tag{2}$$
   (a) Evaluate this limit by letting x get closer and closer to zero. What is $\sin(1/x)$ if:
      i. $x = 0.1$
      ii. $x = 0.01$
      iii. $x = 0.001$

   (b) Make a conjecture for the value of the limit.

   (c) BTW, what would happen if you plugged in $x = 0$?

3. You should have found quite different behavior for the two limits. Why is this? Plot the two functions near $x = 0$. What do you see?

Part II: Adding Sine Waves

1. Write down the equation for a sine wave with a period of 1. Plot this function. Call this function $f(t)$.

2. Write down the equation for a sine wave with a period of 1.01. Plot this function. Call this function $g(t)$.

3. Plot $f(t) + g(t)$—the two sine waves added together. Look at the resulting plot on different scales. What do you notice? Why does the graph have the shape that it does?

Part III: Tangent Lines and Slopes

1. Consider the function $f(x) = x^2$. Determine the value of $f'(3)$. (You can do so numerically or using algebra.)

2. Determine the equation of the line tangent to $f(x)$ at $x = 3$. This may take a little cogitation, as it's something we haven't done yet.

3. Plot $f(x)$ and the tangent line together on the same axes. Does it look like you'd expect it to?

4. Zoom in on the plot near $x = 3$ until the tangent line and $f(x)$ are almost indistinguishable. Does it look like you'd expect it to?

Part IV: The Mathematics of Coffee Cups

In this exercise you will think about how the height h of the coffee in a mug depends on the volume V of the coffee in the mug. For each mug, make a qualitatively accurate sketch of h vs. V.

Figure 1: On the left is a normal-sized mug. Its sides are straight. On the right is a mug that is shorter and wider.

Figure 2: This mug is one of my mugs at home. It is wider on the top than the bottom, and it has straight (but not vertical) sides.

Figure 3: This mug is rounded. It is wider in the middle and narrower on the top and bottom.

Figure 4: This is a "classic diner mug." It is narrower in the middle and wider on the top and bottom.