Lab 09
Calculus I

7 November 2022, College of the Atlantic

- Please work through these problems, in groups if you wish.
- These two problems will be on this week's homework assignment. You should hand in this these problems as part of your homework assignment.

Problem One:

You and your friend both traveled to Des Moines, Iowa, for a fun-filled weekend get together. Monday moring comes and you head your separate ways. You drive due north toward Minneapolis at 100 km/hr, and your friend drives due west toward Omaha at 50 km/hr. As you head northward, you think about how much you miss your friend, and you picture them heading west, getting ever farther away from you. You begin to wonder: how fast is the distance between me and my friend growing?

1. How fast is the distance between you and your friend growing after you have each been driving for one hour?

2. How fast is the distance between you and your friend growing after you have each been driving for two hours?

Problem Two

It is winter break and you are bored. You decide to pass the time by watching cars drive by on a lonely road near your home. The road happens to run directly east-west. You position yourself 5 meters south of the road.

A car appears in the distance to your right. You watch it as it gets closer and closer to you. It passes directly in front of you and then recedes to the left. Eventually it is so far away that you can no longer see it.

This scene repeats itself several times over the next hour. You begin to wonder, how is the speed at which I have to move my head related to how fast the car is going and the position of the car?

Let θ refer to the angle between your line of sight and due east. So if you are looking due east, $\theta = 0$, and if you are looking due north, $\theta = \pi/2$. On this road cars always travel at a constant speed of 60 km/hr.

1. You are watching a car. At the moment in time at which $\theta = \pi/3$, at what rate (in radians/sec) must you turn your head so that the car remains in the center of your line of sight?

By the way, it is possible that you'll need to take the deriviative of a tangent function. Since

$$\tan(\theta) \;=\; \frac{\sin(\theta)}{\cos(\theta)}\;, \tag{1}$$

once can find the derivaitve by using the chain and product[1] rules. Doing so, and using the identiy $\cos^2(\theta) + \sin^2(\theta)$, one arrives at:

$$\frac{d}{d\theta}\tan(\theta) \;=\; \frac{1}{\cos^2(\theta)}\;. \tag{2}$$

[1] Or the quotient rule, if you're one of *those* people