

Calculus I ~~Exam~~ Final Review Assignment

Due November 18, 2022

Thoughts on how to approach this assignment

- This is not an exam! But if there was a final exam for the course, this is what an exam would be like. These are problems that I think should be doable by hand (i.e. without Wolfram Alpha or Desmos) in a reasonable amount of time, and which cover some of the basic skills and ideas from the course.
- Here's a suggestion for how to do this assignment:
 1. Try these problems on your own, without notes. See how far you get.
 2. Then consult your notes and past homework assignments, and see how far you get.
 3. Then, if you have any questions, come to a help session and/or work with others in the class.
- That said, this is a normal homework assignment, in the sense that you are welcome to work together and you can get help from me and any of the TAs.

1. What is the period of $g(x) = -3 \sin(2\pi x) + 5$?
2. Solve for t : $6^{t-1} = 300$. Use logarithms and show your work.
3. A rabbit has been infected with worms. A drug is administered to the ailing rabbit that causes the worm infection load, measured in mg of worm mass, to decrease by 4% every hour. At 8am on Tuesday morning the rabbit's worm load is 75 mg.
 - (a) Write an equation for $W(t)$, the rabbits' worm load t hours after 8am Tuesday.
 - (b) Sketch a graph of $W(t)$. Be sure to label the axes and any intercept(s).
 - (c) When will the rabbit's worm load be approximately 25 mg?

4. (a) Sketch a function that has a positive first derivative for $x < 0$ and a negative first derivative for $x > 0$.
- (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x - 3)$ and $f(x) - 3$. Make it clear which function is which.
- (c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 4a that is invertible? Why or why not?
5. Let $F(T)$ be the cost of heating your house, in dollars per day, when the average outside temperature is T Celsius degrees.
- (a) Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)
- (b) What is the meaning of $F(4) = 3.20$?
- (c) What is the meaning of $F^{-1}(5) = -2.5$?
- (d) What are the units of $F'(T)$?
- (e) In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.

6. For each of the graphs in Fig. 1, find a possible formula for the function.

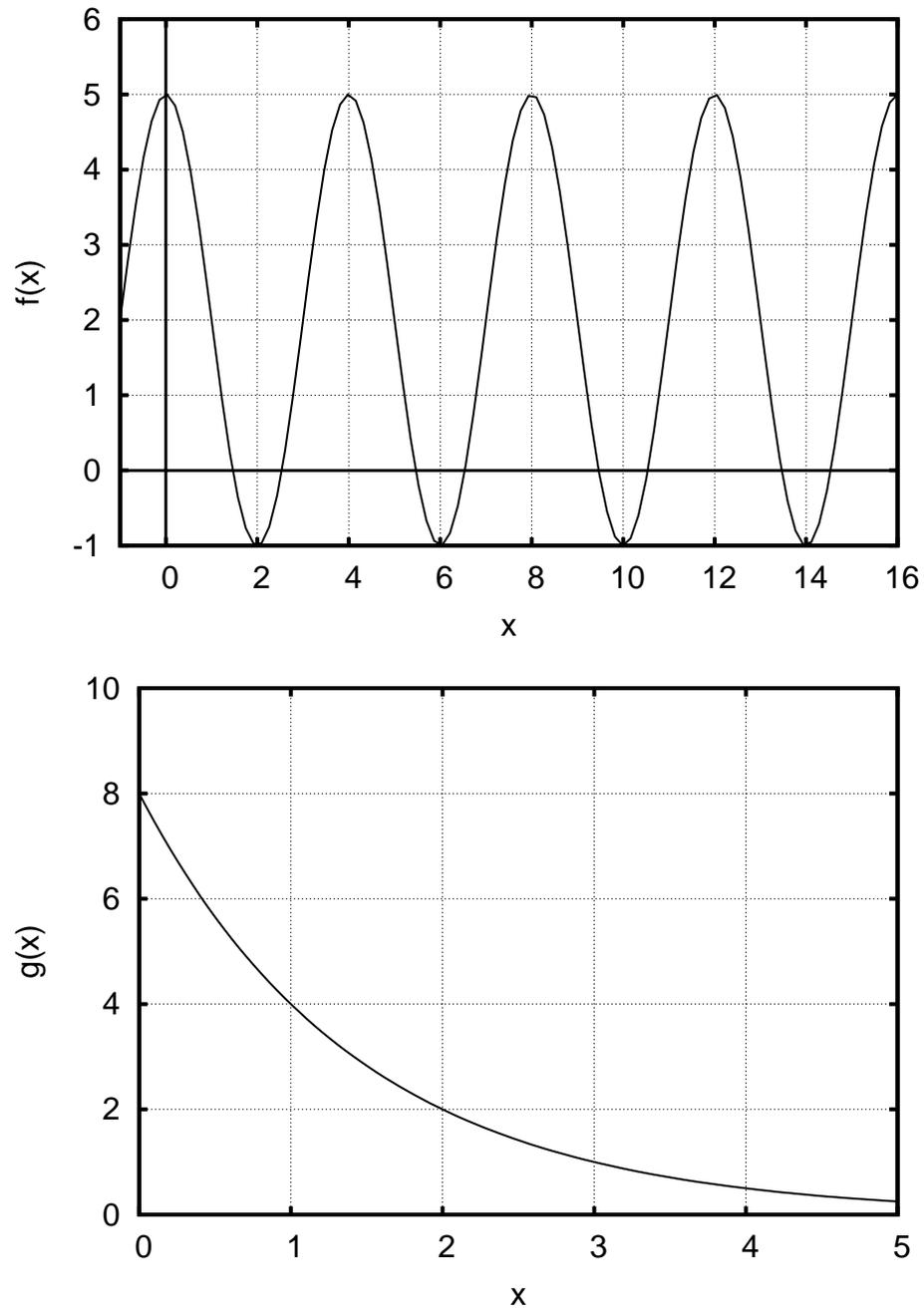


Figure 1: Graphs for problem 6.

7. Let $C(q)$ be the cost, in euros, of q kilograms of organic tofu. Suppose that $C(20) = 50$ and that $C'(20) = 1.3$.
- (a) What are the units of $C'(q)$?
 - (b) What is the practical meaning of $C'(20) = 1.3$?
 - (c) Estimate $C(18)$.

8. Estimate $f'(1.5)$ for the function shown below in Fig. 8. Please show your work.

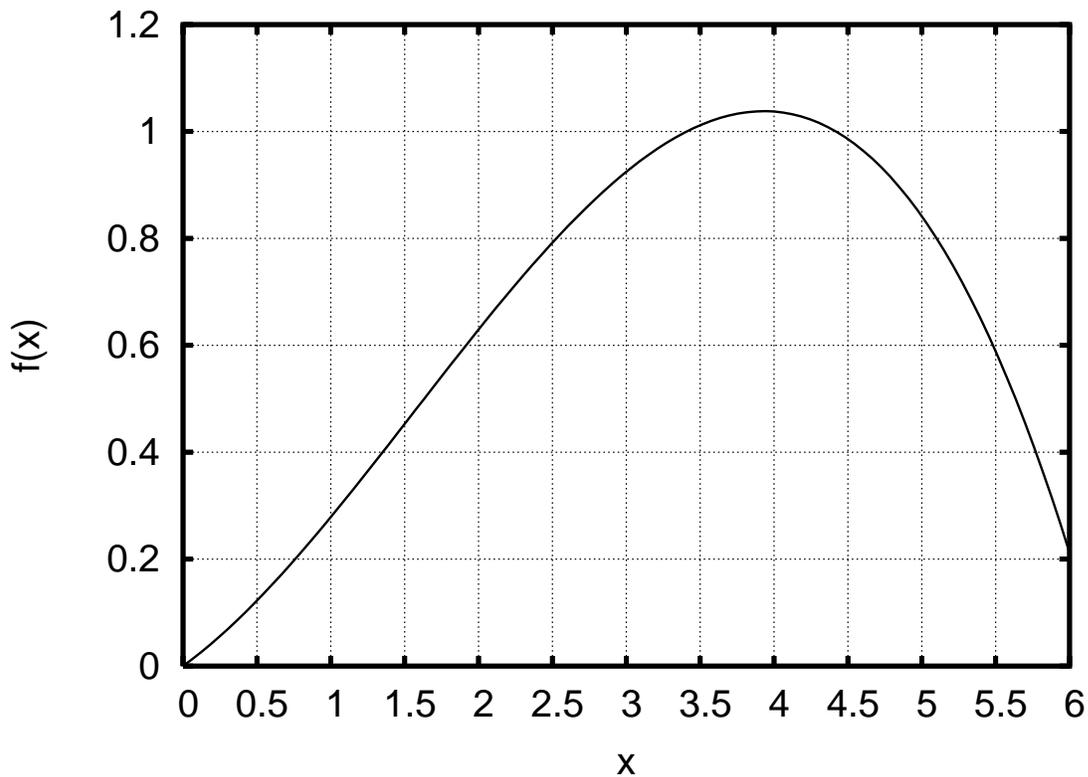


Figure 2: Graph for problem 8.

9. Answer the following questions for the function in Fig. 3. Briefly explain or illustrate your answer.

- (a) Which is larger $f(2)$ or $f(4)$?
- (b) Which is larger $f(4.5)$ or $f(5.5)$?
- (c) Which is larger $f'(2)$ or $f'(4)$?
- (d) Which is larger $f''(2)$ or $f''(3)$?

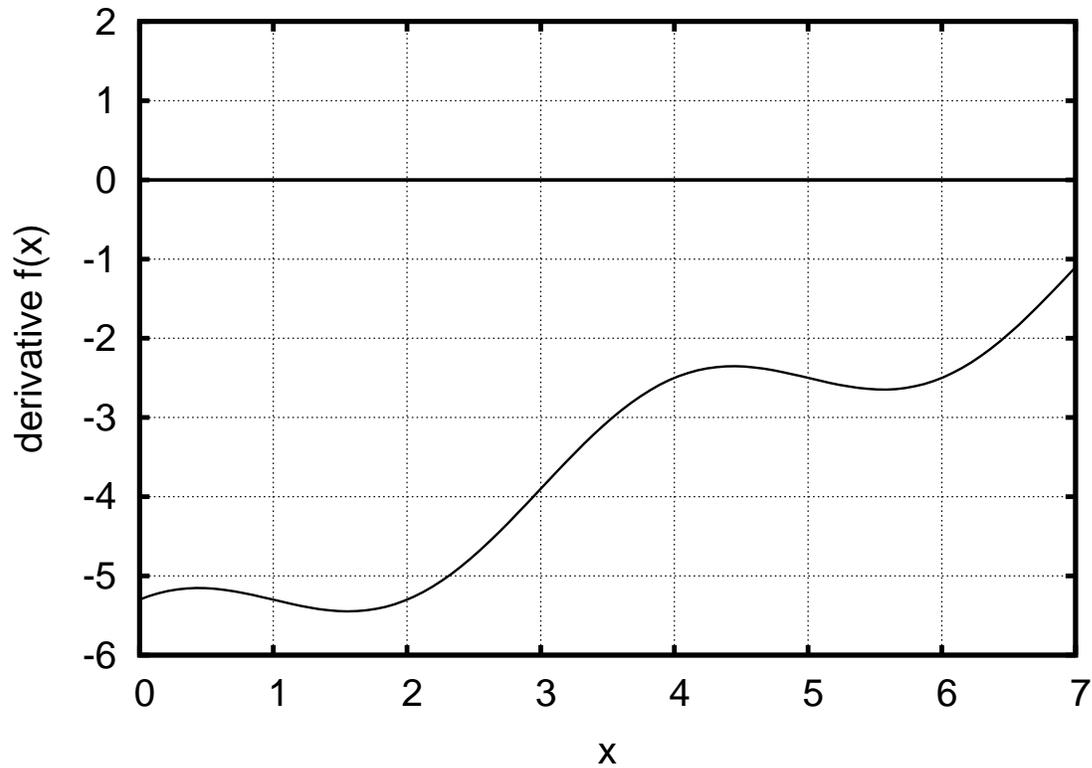


Figure 3: Graph for problem 9. Note that this is a plot of $f'(x)$, not $f(x)$.

10. (a) Make a qualitatively accurate sketches of $h'(x)$ and $h''(x)$ for the function $h(x)$ shown in Fig. 4. Please make the sketches on separate axes, one above the other.
- (b) For approximately what range(s) of x values is $h(x)$ concave down?

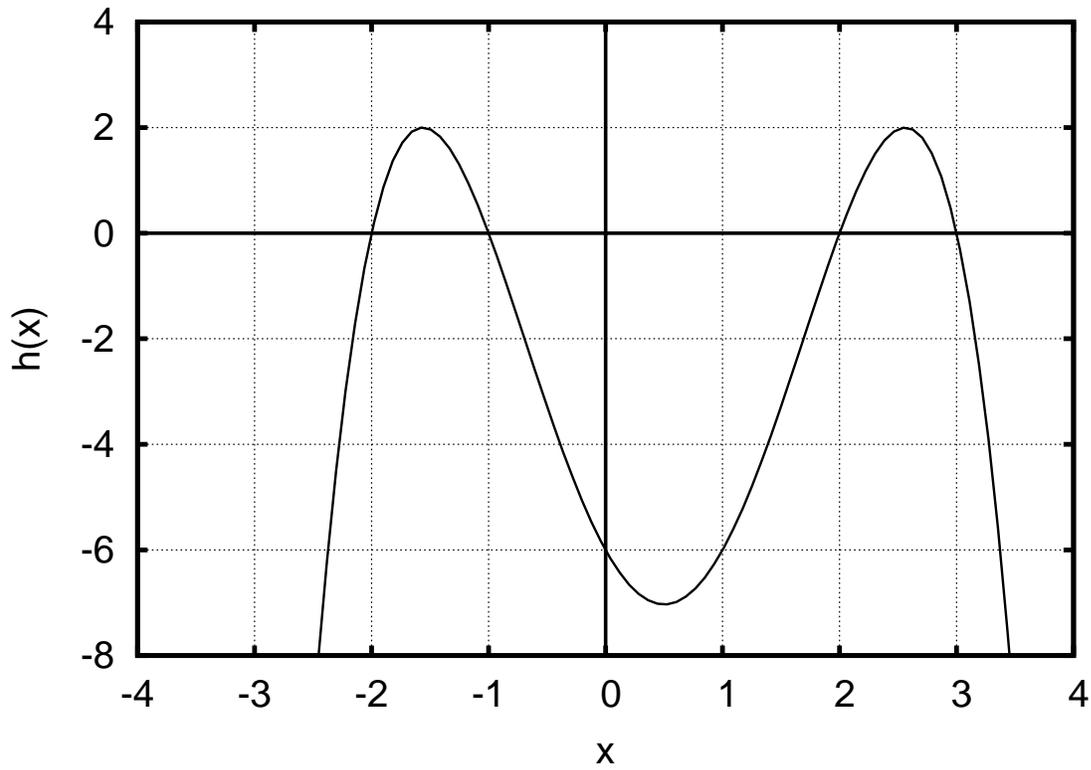


Figure 4: Graph for problem 10.

11. Let $f(10) = 5$, $g(10) = 3$, $f(4) = 2$, $g(4) = 10$, $f'(10) = \frac{1}{3}$, $g'(10) = 4$, $f'(4) = 7$, $g'(4) = -4$. Let $h(x) = 2f(x)g(x)$, and $w(x) = f(g(x))$.

(a) Find $h(10)$.

(b) Find $h'(10)$.

(c) Find $w(4)$.

(d) Find $w'(4)$.

12. Find the derivative of the following functions:

(a) $f(x) = 613 + (5 - 3x^2)^{12}$

(b) $f(x) = \sin(\pi x^2)$

(c) $f(x) = 2^x \cos(3x)$

(d) $f(x) = \frac{3}{x^2} + 7 + 2 \ln(3x)$

(e) $f(x) = e^{-5x^2}$

13. Find the equation of the line tangent to $f(x) = x^2 + 3$ at $x = 2$.

14. Let $g(x) = x^3 - 3x^2 + 17$.

(a) Find and classify all critical points of $g(x)$. Determine x and y values exactly. Don't just estimate them from a graph.

(b) For what values of x is $g(x)$ concave down? Solve for x exactly—don't

15. Let $C(q)$ give the cost, in dollars, of making q donuts. Suppose that $C(50) = 100$ and $C'(50) = 1.1$.

(a) How is $A(q)$, the average cost per donut if q donuts are made, related to $C(q)$. Your answer should be a formula.

(b) What is $A(50)$? (Answer should be a number.)

(c) What is $A'(5)$? (Answer should be a number.)