Lab 05

Calculus I

14 October 2024, College of the Atlantic

- Please work in groups of two or three
- Please check in with me or one of the TAs as you complete each part.
- Please write your answers on this sheet, make a scan of it as a pdf, and upload it google classroom at the end of lab. This assignment is not graded.

Names: _____

Part I: Limits

In this exercise you'll explore the idea of a limit outside of the context of derivatives.
You may wonder why, when evaluating derivatives numerically, we need to consider a series of smaller and smaller h values. Why not just plug in, say $h = 0.0001$ right away? This exercise will show you why...

1. First, we'll explore

$$\lim_{x \to 0} \frac{\sin(x)}{x} \, . \tag{1}$$

   (a) Evaluate this limit by letting x get closer and closer to zero. What is $\frac{\sin(x)}{x}$ if:

      i. $x = 0.1$

      ii. $x = 0.01$

      iii. $x = 0.001$

   (b) Make a conjecture for the value of the limit.

   (c) BTW, what would happen if you plugged in $x = 0$?

2. Next, we'll explore

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) . \tag{2}$$

(a) Evaluate this limit by letting x get closer and closer to zero. What is $\sin(1/x)$ if:

i. $x = 0.1$

ii. $x = 0.01$

iii. $x = 0.001$

(b) Make a conjecture for the value of the limit.

(c) BTW, what would happen if you plugged in $x = 0$?

3. You should have found quite different behavior for the two limits. Why is this? Plot the two functions near $x = 0$. What do you see?

Part II: Tangent Lines and Slopes

1. Consider the function $f(x) = x^2$. Determine the value of $f'(3)$. (You can do so numerically or using algebra.)

2. Determine the equation of the line tangent to $f(x)$ at $x = 3$. This may take a little cogitation, as it's something we haven't done yet.

3. Plot $f(x)$ and the tangent line together on the same axes. Does it look like you'd expect it to?

4. Zoom in on the plot near $x = 3$ until the tangent line and $f(x)$ are almost indistinguishable. Does it look like you'd expect it to?

Part III: Interpreting Derivatives

1. Let $f(t)$ be the number of inches of rain that has fallen since midnight, where t is the time in hours. Interpret the following in practical terms, giving units.

   (a) $f(10) = 1.4$

   (b) $f'(1) = 0.1$

   (c) $f''(10) = -0.2$

   (d) $f^{-1}(1) = 3$

   (e) $(f^{-1})'(1.4) = 3$

Part IV: The Second Derivative

1. A function (not its derivative) is plotted in Fig. 1.

   (a) For what values of x is $f(x)$ positive?

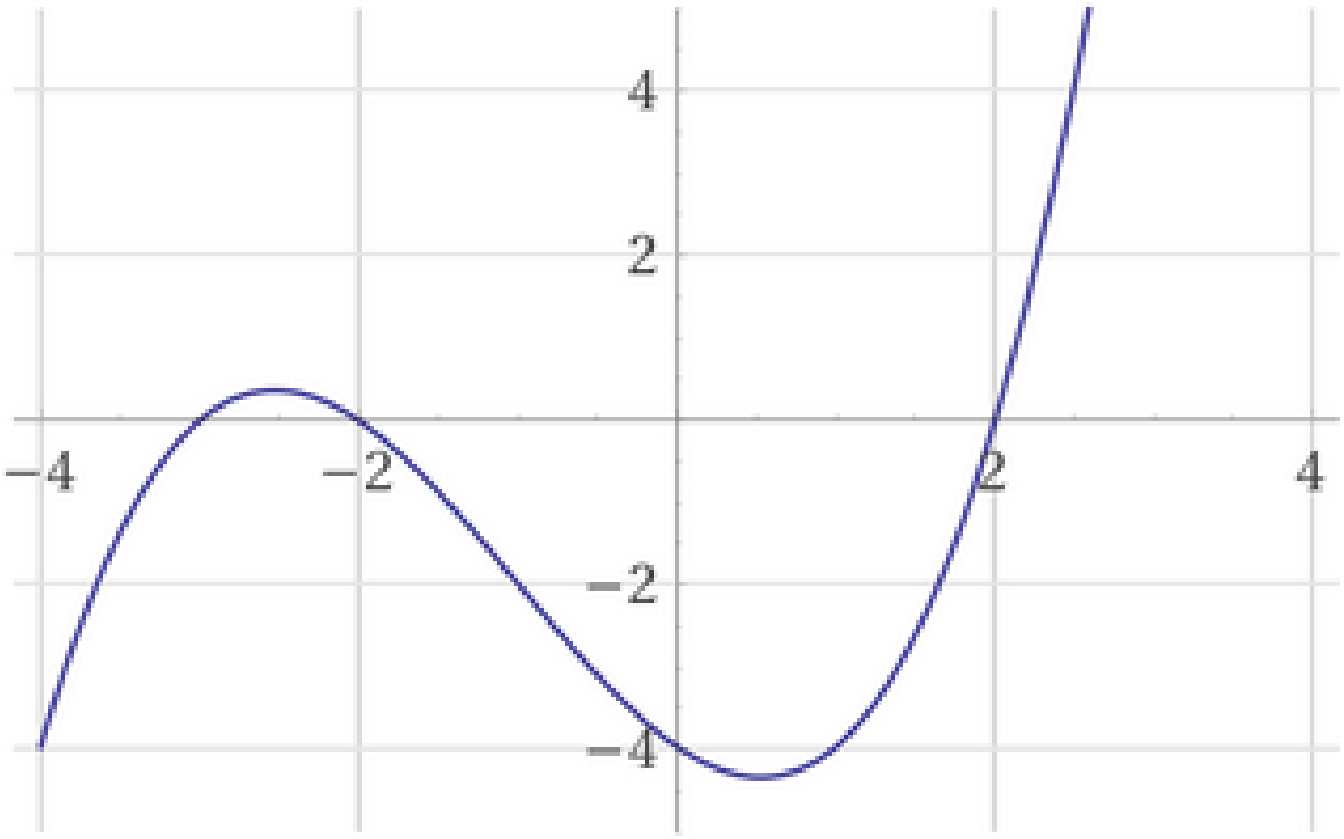
   (b) For what values of x is $f(x)$ negative?

   (c) For what values of x is $f'(x)$ positive?

   (d) For what values of x is $f'(x)$ negative?

   (e) For what values of x is $f''(x)$ positive?

   (f) For what values of x is $f''(x)$ negative?



2. The plot below is of $f'(x)$. For what values of x is:

   (a) $f(x)$ increasing?

   (b) $f(x)$ decreasing?

   (c) $f'(x)$ positive?

   (d) $f'(x)$ negative?

   (e) $f''(x)$ positive?

   (f) $f''(x)$ negative?

3. Sketch $f''(x)$, given the $f'(x)$ in Fig. 5.

4. Sketch a possible $f(x)$ that corresponds to the $f'(x)$ in Fig. 5.

5. Sketch another possible $f(x)$ that corresponds to the $f'(x)$ in Fig. 5.