Lab 07

Calculus I

28 October 2024, College of the Atlantic

- Please work in groups of two or three
- Please write your answers on this sheet, make a scan of it as a pdf, and upload it google classroom at the end of lab. Thanks.

Names: _____

Recall that in the lab last week we saw that the temperature $T(t)$ of an object initially at T_0 when placed in an environment of temperature T_{env} is described by the following equation:

$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}})e^{-kt} , \tag{1}$$

where t is the time since the object was placed in the environment and k is a constant that is related to the rate of cooling. Equation (1) can also be written as:

$$T(t) = b + ae^{-kt} . \tag{2}$$

Part I: Newton's Law of Cooling

Newton's law of cooling is:

$$\frac{dT}{dt} = -k(T(t) - T_{\text{env}}) . \tag{3}$$

1. Translate Eq. (3) into words. In one or two sentences, what is Eq. (3) telling us?

2. Equation (3) is an example of a differential equation. Before we get to differential equations, let's think about regular equations. For example, here's a regular equation:

   $$x^3 - 4x = -6 . \tag{4}$$

   This is a statement that is not true in general; it is only true for particular value(s) of x.

   It is hard to solve this equation. But it is not so hard to check to see if a particular x is a solution. Just plug in and see if the left-hand and right-hand sides are equal.

(a) Is -1 a solution to Eq. (4)?

(b) Is 2.525 a solution to Eq. (4)?

3. Now let's go back to the differential equation, Eq. (3). This equation is not true in general; it is only true for particular function(s) $T(t)$.

   It is hard to solve this equation. But it is not so hard to check to see if a particular $T(t)$ is a solution. Just plug in and see if the left-hand and right-hand sides are equal.

   (a) Show that $T(t) = -t^2$ is not a solution to Eq. (3).

   (b) Show that the $T(t)$ given by Eq. (2) is a solution to Eq. (3).

Part II: A Mystery in Bar Harbor

This fictional story is based on a scenario presented on pp. 194–5 of *Conversational Calculus*, by David W. Cohen and James M. Henle, Addison Wesley, 1997. Here is the scenario.

You are walking to Calculus class on a Thursday morning. You notice an incoming call on your cell phone: it's Bar Harbor police chief Jim Willis. That's odd... why would he be calling you? It might be important, so you take the call.

Chief Willis says hello and tells you that there is a tragic situation and he thinks you might be able to help.

"I'm happy to help if I can," you reply.

"Thanks," Willis replies. "Here's the situation: A man was found dead in the vault of the Bank of Bar Harbor this morning. For now we're referring to him as John Doe, since his family hasn't been notified. He died from a single bullet wound, and there was typed suicide note found next to the body. It could be a suicide, but we suspect there might be foul play. Doe was in excellent health and had no financial problems. Doe worked at the bank, and apparently he shared on social media that he thought his boss, bank president Penny Max, might try to kill him."

"Have you talked to Penny?" you ask.

"Yes," the Chief replies. "She denies everything. Ms. Max closed the bank vault last night at 5:00pm, time-locking it so that nobody could open it until 8:00am today. We've looked at data logs and have verified that this is indeed the case. Nobody entered or exited the vault between 5:00pm last night and 8:00am today. Ms. Max discovered the body this morning when she opened the vault. She says that she didn't noticed anyone in the vault last night when she locked it, but that it would be possible that someone was hiding unseen in the vault. We think Penny Max may be guilty, but we're not sure. We need more evidence."

"Ummm .... ok. But why are you calling *me*?" you ask.

"I left a voicemail with the FBI," Chief Willis responds. "And then I remembered reading on twitter that you're taking Calculus, and that Calculus is supposed to be super useful. I don't know... I just didn't know who else to turn to."

"Well, thank you for thinking of me, but I'm not sure I can help," you reply. But then something occurs to you. "What temperature is the body?" you ask.

"Mr. Doe was 65.3 degrees at 8:40 this morning," says Willis.

"What temperature is John Doe now?"

"Let me check." You hear Willis ask an officer to take another temperature reading. "Doe is now 65.1 degrees." You glance at your phone and note that it is 10:37am.

"One more question: what temperature is the vault kept at?"

You hear Willis talking with one of the bank officials. After a moment, he comes back on the line. "The vault is kept at a constant temperature of 65 degrees by a modern and reliable heat pump. The bank used to use oil for heat, and they kept the vault at 70 degrees. A group of slightly self-righteous but very helpful COA students convinced them that 70 was too hot, and pointed us toward a nice rebate program that enabled us to buy the heat pump. Anyway, the vault is always at 65 degrees."

"Thanks," you respond. "I now have enough information to figure out what has happened. I'll need to do a few calculations, but I'll get back to you soon."

"Thank you," say the Chief. "Whatever happened this is a tragic situation. I'd really like to find the truth, so that justice and healing can begin."

1. Solve the mystery. Was it a murder, or was it a suicide? Show your work—the details of your calculations might need to referred to in a trial.