EXAM 1

13 February 2003

Directions

- **You may not collaborate on this exam; do not work with others.**
- This exam is open notes, open book. This exam is untimed, but unless I hear otherwise, I expect you to finish sometime Friday.
- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.
- To receive full credit on most of these problems you must show your work clearly.

1. In October 2002, the rat population in a dorm is 15. The population grows quickly until March 2003. Then, in April 2003, the population crashes suddenly. In May, the population starts growing again. In June, the population is 50.
   (a) Sketch a possible graph for the population $P(t)$ of the rat population as a function of time t, measured in months since October 2002.
   (b) Sketch $P'(t)$.
   (c) Is $P(t)$ invertible? Why or why not?
   (d) What are the units of $P'(t)$?
   (e) What is the meaning of the statement $P(3) = 35$?
   (f) Suppose $P'(2) = 5$. In practical terms, what is this telling you?
2. Find the derivative of $g(x) = \log x$ at $x = 4$. To do this, you'll need to numerically evaluate the limit. (10 points)
3. Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x) = 3x^2 - x$.
4. The number of bacteria in milk grows at a rate of 10% a day once the milk has been bottled. When the milk is put in the bottles, it has an average bacteria count of 500 million per bottle.

(a) Write an equation for $f(t)$, the number of bacteria t days after the milk is bottled.

(b) Sketch a graph of the number of bacteria against time. Be sure to label the axes and any intercept(s) and/or any asymptote(s).

(c) Suppose milk cannot be safely consumed if the bacteria count is greater than 3 billion per bottle. How many days will the milk be safe to drink once it has been bottled?

5. Let $h(t)$ be the height of a student's eyebrows, measured in centimeters from the bottom of her or his nose, as a function of time t. Let t be measured in days since January 2, 2003.

(a) What is the meaning of the statement $h(4) = 5$?

(b) What are the units of $h'(t)$?

(c) **Optional:** What can you say about the sign of $h'(t)$? Why?

6. Consider the following table of values for a function $J(x)$. (10 points)

x	$J(x)$
0.0	1.0000
0.1	0.9975
0.2	0.9900
0.3	0.9776
0.4	0.9604
0.5	0.9385
0.6	0.9120
0.7	0.8812
0.8	0.8462
0.9	0.8075
1.0	0.7652

(a) Estimate $f'(5)$.

(b) What is the average rate of change of $J(x)$ between $x = 0.2$ and $x = 0.6$?

7. (a) Sketch a function that has a negative first derivative for $x < 3$, a derivative of zero for $3 < x < 8$ and a positive first derivative for $x > 8$.

   (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x-2)$ and $f(x) - 2$. Make it clear which function is which.

   (c) Is your $f(x)$ invertible? Why or why not? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 7a that is invertible? Why or why not?

8. For each of the four graphs in Fig. 1, find a possible formula for the function.

9. The following questions all refer to the graph in Fig. 2. As you answer these questions, you may wish to make several additional sketches of this figure. Please label things clearly.

   (a) Show how to represent the quantity $\frac{f(4)-f(2)}{4-2}$ on the graph.

   (b) Which is larger, $f(2)$ or $f(4)$? Explain.

   (c) Which is larger, $f'(2)$ or $f'(4)$? Explain.

   (d) Which is larger, $f'(2)$ or $\frac{f(4)-f(2)}{4-2}$? Explain.

10. For the function shown in Fig. 3:

    (a) Estimate $g'(0.5)$ and $g'(3.5)$

    (b) Sketch $g'(x)$.

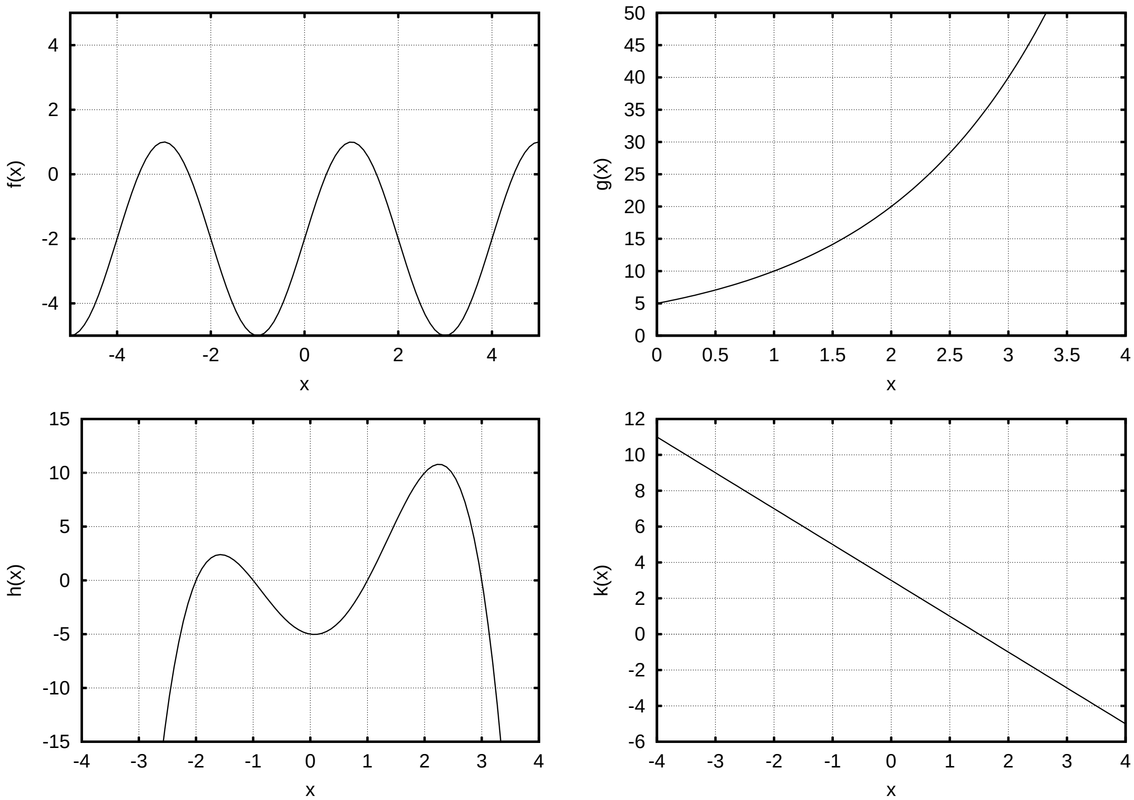

Figure 1: Functions for Problem 8

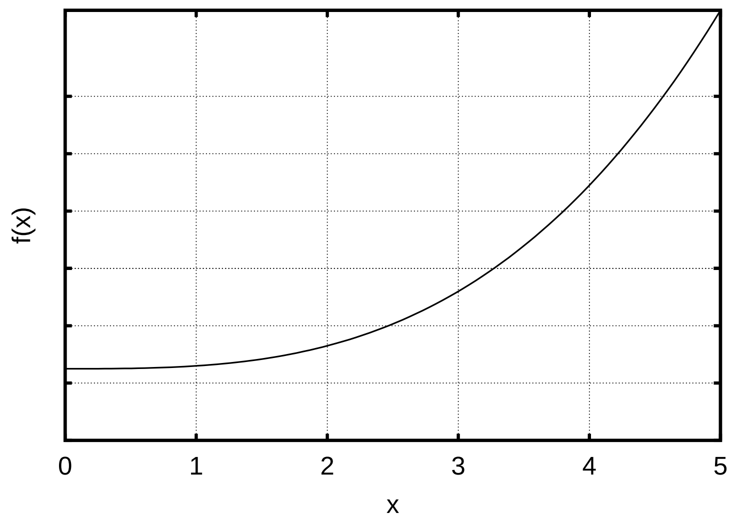

Figure 2: Function for Problem 9

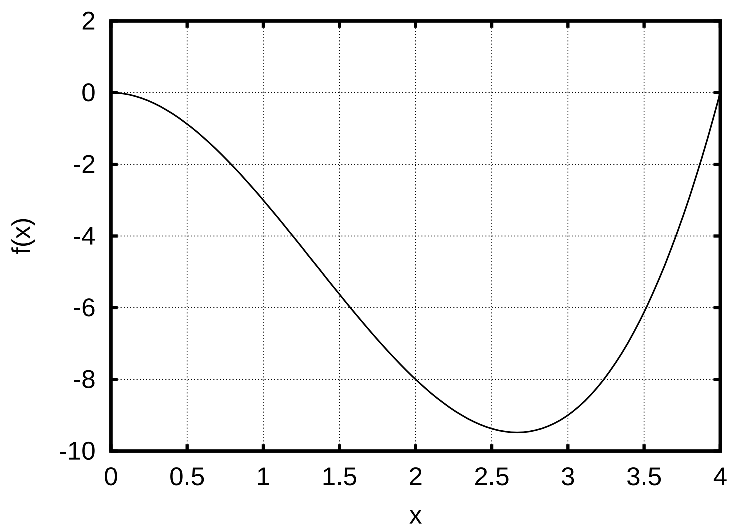

Figure 3: Function for Problem 10