EXAM 2

Winter 2003

Directions

- **You may not collaborate on this exam; do not work with others.**
- This exam is open notes, open book. This exam is untimed, but unless I hear otherwise, I expect you to finish by 5:30 pm Friday March 14.
- When you are done with the exam, give it to me or put it under my door. Don't put it my mailbox.
- To receive full credit on most of these problems you must show your work clearly.

1. Determine upper and lower estimates for the following integral:

$$\int_0^3 2^{-x}\,dx\ . \tag{1}$$

   Do not use the fundamental theorem to get your estimate. Please show your work so that your method is clear.

2. Fig. 1 shows the rate $r(t)$, in gallons per hour, at which maple syrup is leaking from a large tanker. The time t is measured in hours.

   (a) Write down an integral that represents the total amount of maple syrup that leaks in the first seven hours.

   (b) Approximate this integral. Briefly explain your method.

   (c) Approximate the average leak rate during the first seven hours.

3. Let $f(10) = 5$, $g(10) = 3$, $f(4) = 2$, $g(4) = 10$, $f'(10) = \frac{1}{3}$, $g'(10) = 4$, $f'(4) = 7$, $g'(4) = -4$. If $h(x) = 2f(x)g(x)$, and $w(x) = f(g(x))$.

   (a) Find $h(10)$.

   (b) Find $h'(10)$.

   (c) Find $g(4)$.

   (d) Find $g'(4)$.

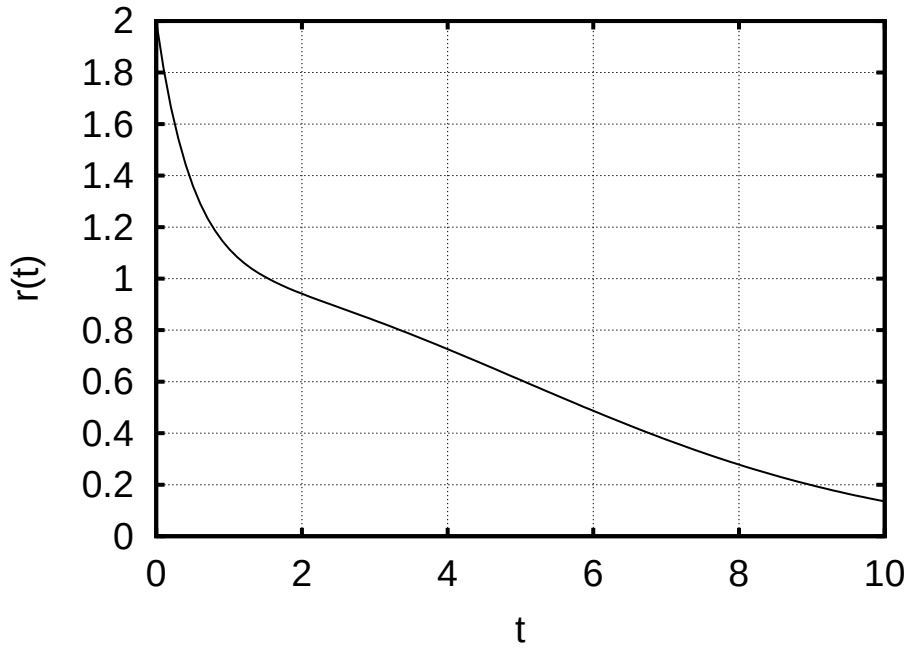

Figure 1: The rate $r(t)$ of maple syrup leakage.

4. Find the derivative of the following functions:

   (a) $f(x) = 99 + (x+4)^{50}$

   (b) $f(x) = \sin(x^2)$

   (c) $f(x) = \sin(e^{2x})$

   (d) $f(x) = 3^{2x}\sin(2x)$

   (e) $f(x) = \arctan(x^2)$

   (f) $f(x) = x^2 + 3 + 2\ln(3x)$

5. What is the 25^{th} derivative of $f(x) = e^{3x}$?