Calculus I Midterm

10 February 2005

Directions

- **You may not collaborate on this exam; do not work with others.**

- You may consult your notes, your textbook, or any other written material while doing this test. You may use calculators and computer programs.

- This exam is untimed, but unless I hear to the contrary, I expect you to finish sometime Friday.

- When you are done with the exam, give it to me or slip it under my office door. Don't put it my mailbox.

- If you have clarifying questions on the exam, please ask me. Do not ask the TAs.

- To receive full credit on most of these problems you must show your work clearly.

1. Solve for t: $2^t = 75$.

2. Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x) = 3x^2$. **Do not use the power rule or any other shortcut.**

3. A car loses 10% of its value every year. Suppose you buy a new car for $12,000$ euros.

   (a) Write an equation for $V(t)$, the value of the car that is t years old.
   (b) Sketch a graph of $V(t)$ versus time.
   (c) How many years after you buy the car is the car worth half of its initial value?

4. Consider the following table of values for a function $J(x)$. (10 points)

x	$J(x)$
0.0	1.0000
0.1	0.9975
0.2	0.9900
0.3	0.9776
0.4	0.9604
0.5	0.9385
0.6	0.9120
0.7	0.8812
0.8	0.8462
0.9	0.8075
1.0	0.7652

(a) What is the average rate of change between $x = 0.4$ and $x = 0.9$?

(b) Estimate $J'(3)$.

5. (a) Sketch a function that has a negative first derivative for $x < 3$, a derivative of zero for $3 < x < 8$ and a positive first derivative for $x > 8$.

(b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(x - 2)$ and $f(x) - 2$. Make it clear which function is which.

(c) Is your $f(x)$ invertible? Is it possible to come up with an $f(x)$ that satisfies the criteria of question 5a that is invertible? Why or why not?

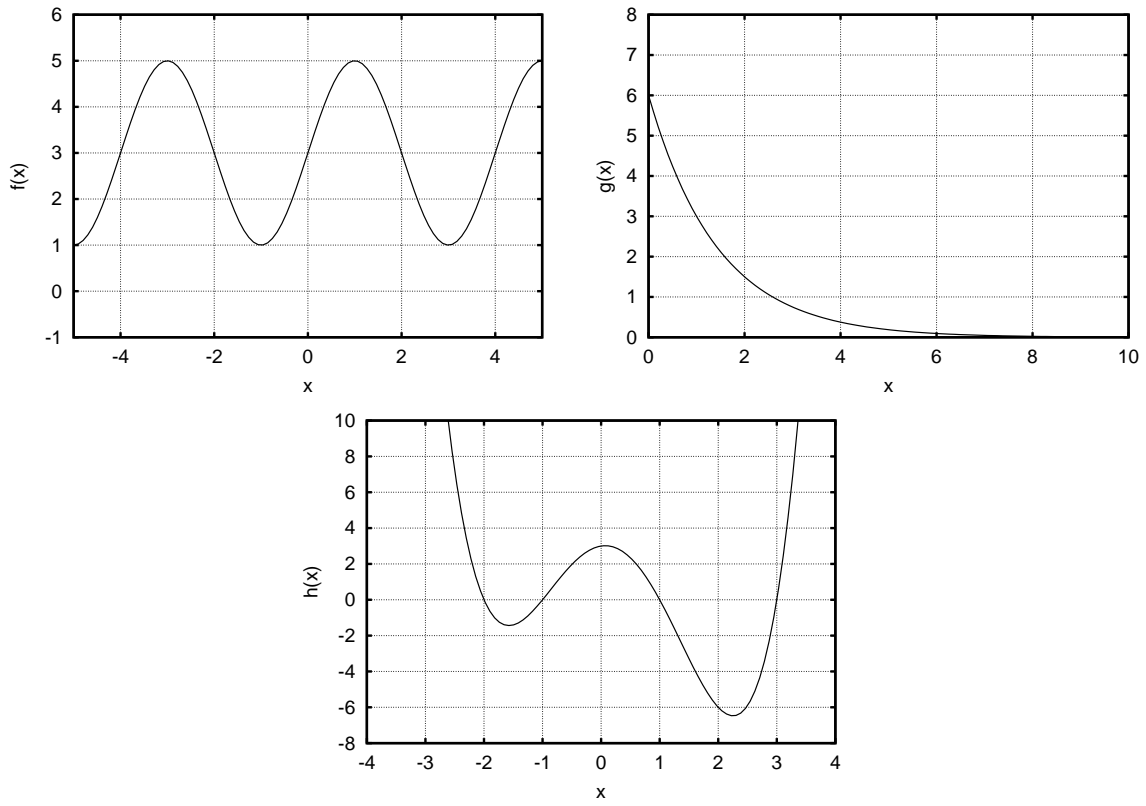6. For each of the three graphs in Fig. 1, find a possible formula for the function.

Figure 1: Graphs for problem 6.

7. Let $F(T)$ be the cost of heating your house, in dollars per day, when the outside temperature is T Celsius degrees.

   (a) Make a rough sketch a possible graph of $F(T)$. (There are many possible answers.)

   (b) In practical terms, what does $F(3) = 1.4$ mean?

   (c) What are the units of $F'(T)$?

   (d) In practical terms what does $F'(-4) = -.17$ mean? Be sure to explain why the minus sign is there.

8. Numerically (i.e., using a calculator and making a table of numbers), estimate the derivative of $g(x) = \log(2x)$ at $x = 3$.

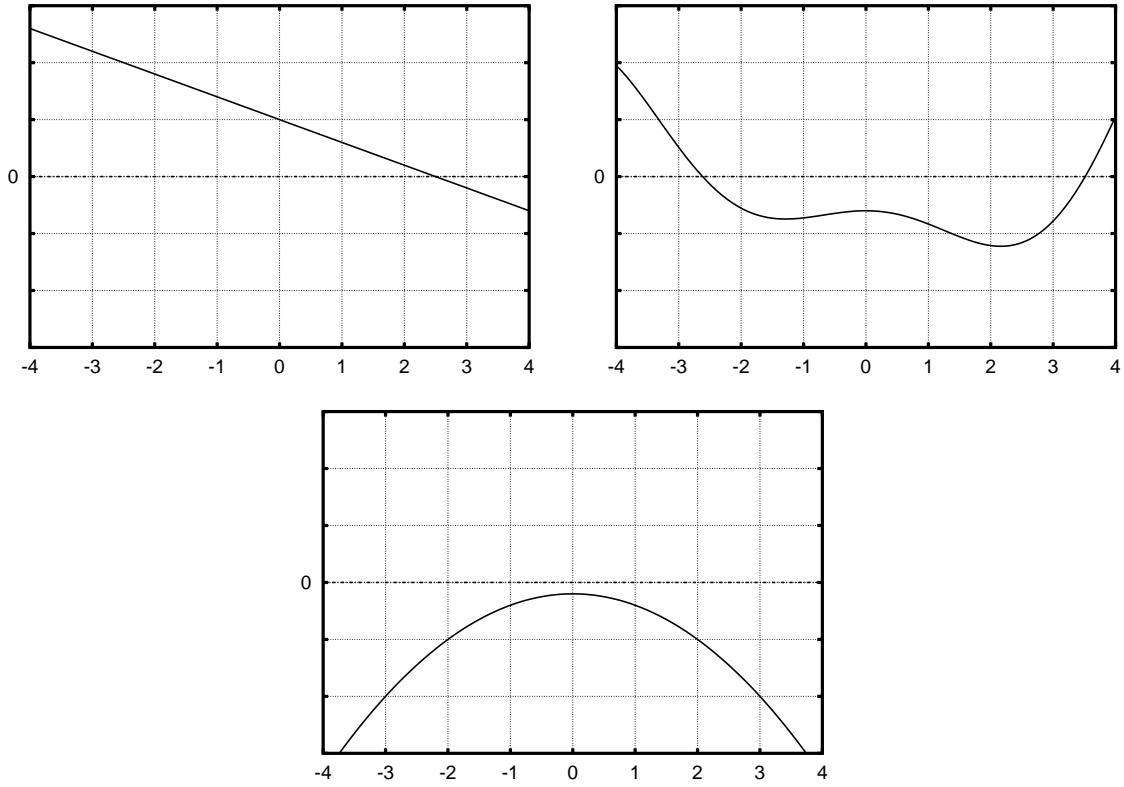9. For each of the functions in Fig. 2, sketch the derivative.

Figure 2: Graphs for problem 9.

10. Suppose we have a function such that $h(40) = 90$ and $h'(40) = -3$. Use this information to estimate $h(43)$.