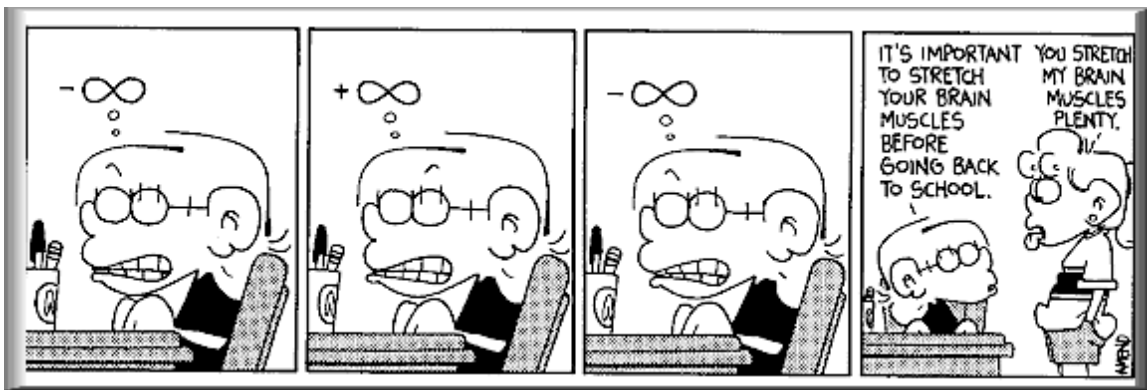# EXAM 1

20 October 2001

Directions

- **You may not collaborate on this exam; do not work with others.**

- This exam is open notes, open book. This exam is untimed, but unless I hear otherwise, I expect you to finish sometime Friday.

- When you are done with the exam, give it to me. Don't put it my mailbox.

- To receive full credit on most of these problems you must show your work clearly.

1. As a result of a series of computer blunders, I managed to delete many of your homework scores. Please write down your scores on homeworks 1 through 4. Thanks. (0 points).

2. Short Answers. (10 Points).

    (a) Which function dominates as $x \to \infty$: $x^{1/3}$ or $\log x$?

    (b) Which function dominates as $x \to \infty$: $x^{1/3}$ or x^{-3}?

    (c) Is $f(x) = 2\sin(2x) + \sin(3x)$ a periodic function? If so, what is its period and why?

    (d) Let $f(x) = e^{2x}$ and $g(x) = \ln x + 3$. What is $f(g(x))$? Simplify if possible.

3. Algebraically (i.e. without making a table of numbers or using a calculator) determine the derivative of $f(x) = 4x - 3x^2$ (10 points).

4. An almanac lists the earth's population in 1980 as 4.478 billion and in 1994 as 5.642 billion. Assume the population is growing exponentially. (15 points)

   (a) Predict the earth's population in the year 2000.

   (b) What is the doubling time of the earth's population?

5. Consider the graph of $f(x)$ below. For each lettered point on the graph, indicate whether f, f', and f'' are positive, negative, or zero. (10 points)

6. (a) Sketch a function that has a positive first derivative for $x < 1$, a negative first derivative for $1 < x < 5$ and a zero first derivative for $x > 5$. (5 points)

   (b) Call this function $f(x)$. On the same axes as your original graph, sketch $f(3x)$ and $3f(x)$. Make it clear which function is which. (5 points)

7. In January 1999, the mongoose population on an island is 1000. The mongoose population on this island grows quickly for three months. Then, in April, the population crashes suddenly due to an epidemic. After the epidemic, starting in May, the population grows again. By the end of July, the population is at 4000. (15 points)

   (a) Sketch a possible graph for the population $P(t)$ of the mongoose population as a function of time t, measured in months since January 1999.

   (b) Sketch $P'(t)$.

   (c) Is $P(t)$ invertible? Why or why not?

   (d) What are the units of $P'(t)$?

   (e) Suppose $P'(5.5) = 500$. In practical terms, what is this telling you?

8. Consider the function shown on the graph below. Sketch the derivative of this function.

9. Find the derivative of $g(x) = \log x$ at $x = 4$. To do this, you'll need to numerically evaluate the limit. (10 points)

10. For each of the four graphs below, find a possible formula for the function. (10 Points)