

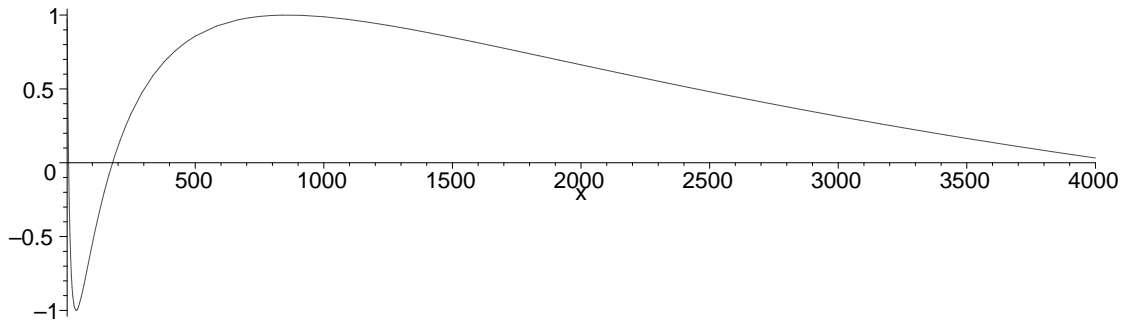
The difference between $f(g(x))$ and $g(f(x))$

$\sin(\log(3x))$ is a very slowly wiggling sine wave:

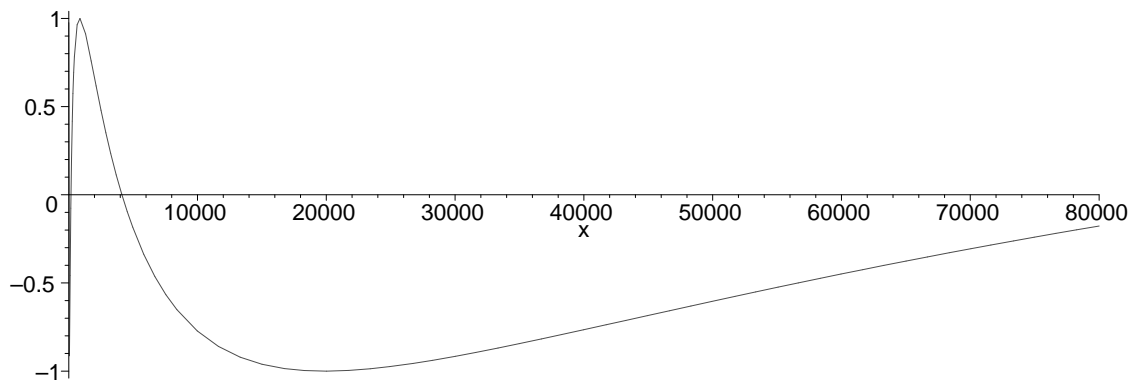
```
> f := x->sin(log(3*x));
```

$$f := x \rightarrow \sin(\log(3x))$$

```
> plot( f(x), x=2..4000);
```



```
> plot( f(x), x=2..80000);
```



```
>
```

```
> diff(f(x),x);
```

$$\frac{\cos(\ln(3x))}{x}$$

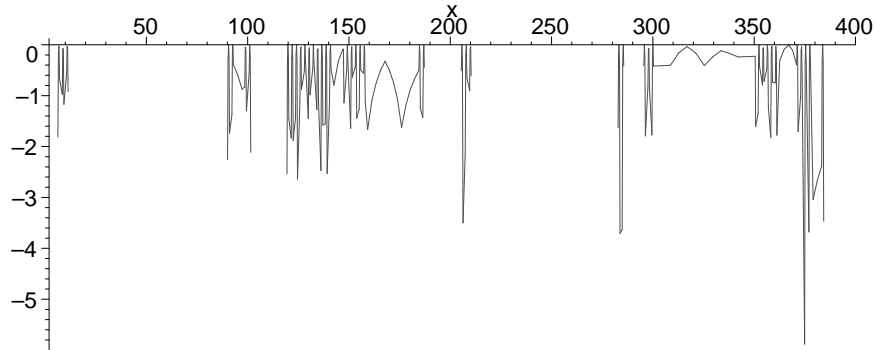
This is a stretched cos function whose amplitude is decaying by $1/x$.

$\sin(\log(x))$ is very different.

```
> g := x -> log(sin(3*x));
```

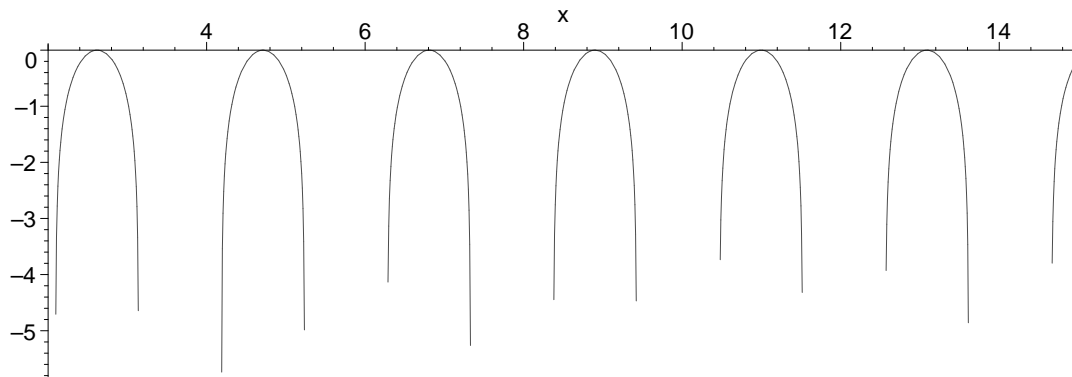
$g := x \rightarrow \log(\sin(3x))$

```
> plot( g(x), x=2..400);
```



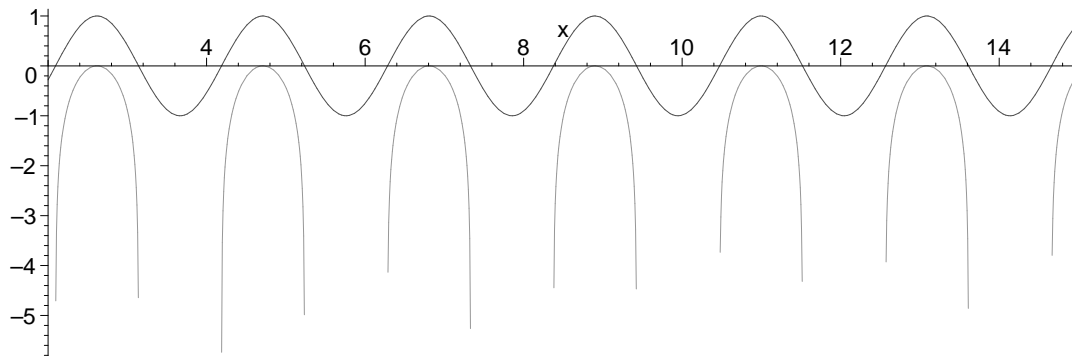
yikes! what's this about? let's try a different range.

```
> plot( g(x), x=2..15);
```



Why is this flying off to negative infinity? What's happening is that the sine function is wiggling between +1 and -1. Then the log function acts on the wiggling. The log of 0 is -infinity, and the log of minus numbers isn't defined. This is why there are gaps in the function. When $\sin(3x)$ is negative, $\log(\sin(3x))$ "disappears".

```
> plot( { g(x), sin(3*x) }, x=2..15);
```



```
>
```