**(b)** A cone with height and radius both equal to $r$ has volume, $V$, proportional to $r^3$; that is, $V = kr^3$ for some constant $k$. A lab experiment is done to measure the volume of several cones; the results are in the following table. Using the method of part (a), determine the best value of $k$. [Note: Since the volumes were determined experimentally, the values may not be accurate. Assume that the radii were measured accurately.]

| Radius (cm) | 2 | 5 | 7 | 8 |
|---|---|---|---|---|
| Volume ($cm^3$) | 8.7 | 140.3 | 355.8 | 539.2 |

**(c)** Using the method of part (a), show that the best fitting line of the form $y = mx$ for the points $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$ has

$$m = \frac{x_1y_1 + x_2y_2 + \cdots + x_ny_n}{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

## 3. Firebreaks

The summer of 2000 was devastating for forests in the western US: over 3.5 million acres of trees were lost to fires, making this the worst fire season in 30 years. This project studies a fire management technique called *firebreaks*, which reduce the damage done by forest fires. A firebreak is a strip where trees have been removed in a forest so that a fire started on one side of the strip will not spread to the other side. Having many firebreaks helps confine a fire to a small area. On the other hand, having too many firebreaks involves removing large swaths of trees.[14]

**(a)** A forest in the shape of a 50 km by 50 km square has firebreaks in rectangular strips 50 km by 0.01 km. The trees between two firebreaks are called a stand of trees. All firebreaks in this forest are parallel to each other and to one edge of the forest, with the first firebreak at the edge of the forest. The firebreaks are evenly spaced throughout the forest. (For example, Figure 4.115 shows four firebreaks.) The total area lost in the case of a fire is the area of the stand of trees in which the fire started plus the area of all the firebreaks.

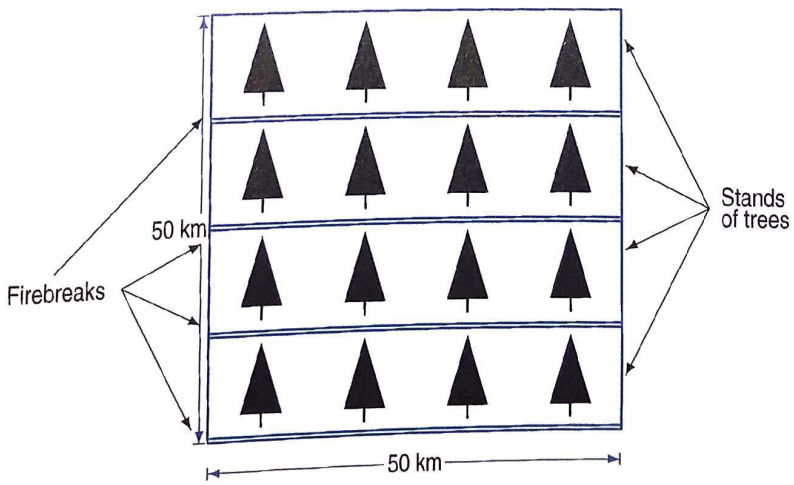


Figure 4.115

(i) Find the number of firebreaks that minimizes the total area lost to the forest in the case of a fire.

(ii) If a firebreak is 50 km by $b$ km, find the optimal number of firebreaks as a function of $b$. If the width, $b$, of a firebreak is quadrupled, how does the optimal number of firebreaks change?

[14] Adapted from D. Quinney and R. Harding, *Calculus Connections* (New York: John Wiley & Sons, 1996).

**(b)** Now suppose firebreaks are arranged in two equally spaced sets of parallel lines, as shown in Figure 4.116. The forest is a 50 km by 50 km square, and each firebreak is a rectangular strip 50 km by 0.01 km. Find the number of firebreaks in each direction that minimizes the total area lost to the forest in the case of a fire.

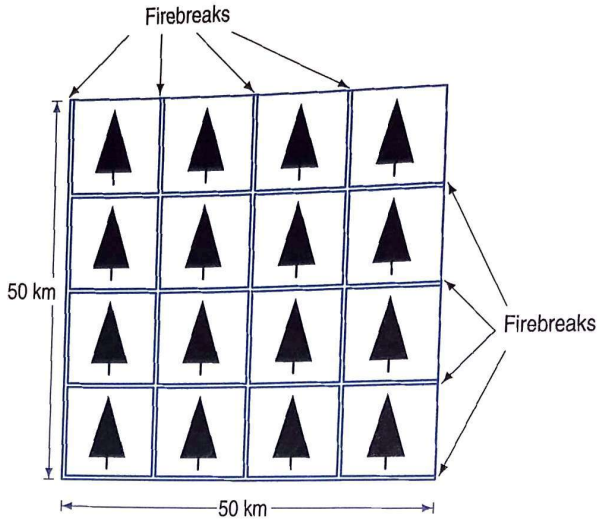


Figure 4.116