Calculus II

Homework Ten

(not really a final)

Due Friday 1 June, 2007

1. The number of email messages per day received by an academic administrator is well approximated by a Gaussian distribution with mean 80 and standard deviation 20.

   (a) Write down the probability density function $p(x)$.

   (b) Determine the probability that the administrator receives between 90 and 110 email messages. Do this two ways:

      i. Have Maple evaluate a definite integral.

      ii. Approximate the definite integral by hand. Form both left and right hand sums. Use a Δx of 10.

   (c) What is the probability that between 60 and 100 messages are received?

   (d) What is the probability that more than 150 messages are received?

2. Let t denote the time it takes between successive emails, measured in minutes. The distribution of t is well described by an exponential distribution:

$$p(t) = \lambda e^{-\lambda t} , \tag{1}$$

   where $t > 0$ and λ is a positive constant.

   (a) Suppose that the mean time between emails is 15 minutes. Use this information to solve for λ.

   (b) Calculate the cumulative distribution function $P(t)$.

   (c) Use $P(t)$ to answer the following questions:

      i. What is the probability that the time between successive emails is less than 5 minutes?

      ii. What is the probability that the time between successive emails is more than one hour?

      iii. What is the probability that the time between successive emails is between 10 and 20 minutes?

3. For the purposes of this problem, assume the following:

   - College costs $100,000$ dollars. Assume that you would pay this amount of money up front. I.e, you need to pay this amount the day you enter college. This isn't true, but it simplifies things.
   - Interest rates are five percent compounded continually.
   - Assume that people save all of the money they make. This, of course, isn't true.
   - If you do not go to college your starting salary will be $\$21,000$ a year.
   - If you do go to college your starting salary will be $\$33,000$ a year.
   - Assume that you get a raise of 4 percent a year. Further, assume (not that realistically) that this raise is awarded continuously instead of at the end of every year. Ignore inflation.

   The above data about salaries was taken from U.S. Census Bureau, "The Big Payoff: Educational Attainment and synthetic Estimates of Work-Life Earnings," July 2002, `www.census.gov/prod/2002pubs/p23-210.pdf`.

   (a) How much money will the following two options yield the time you're thirty?
      - At age 20 you take the $100,000$ that you would have spend on college, put it in the bank, and go to work.
      - You go to college and when you are 25 you start work.

   (b) How much money will the above two options yield at the time you're forty?

   (c) **(Optional)** Women aged $25 - -29$ in the US with a bachelor's degree make, on average 81% what men make. Suppose a man and a women go to college and both start work at 25. How much money will each have when they are 60?