

Theorems about Integrals

The Fundamental Theorem of Calculus

The big idea:

$$\text{Total change in position} = f(b) - f(a) = \int_a^b f'(t) dt . \quad (1)$$

Example: Recall that the total distance traveled by the cat from time $t = 0$ to $t = 4$ was given by:

$$\text{Total distance traveled} = \int_0^4 v(t) dt , \quad (2)$$

where $v(t) = f'(t)$ was the *rate*, in meters per second, at which cat ran.

Properties of Definite Integrals

These properties are all “obvious” if you think of definite integrals as corresponding to areas.

1.
$$\int_a^b f(x) dx = - \int_b^a f(x) dx . \quad (3)$$

2.
$$\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx . \quad (4)$$

3.
$$\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx . \quad (5)$$

4.
$$\int_a^b cf(x) dx = c \int_a^b f(x) dx . \quad (6)$$