Second Fundamental Theorem

The Fresnel Cosine Integral $C(x)$ is defined by

$$C(x) \equiv \int_0^x \cos(\frac{\pi}{2}t^2)\, dt\ . \tag{1}$$

This integral arises in certain optics applications.

1. Plot the integrand, $\cos(\frac{\pi}{2}x^2)$.
2. Using the plot of the integrand, sketch the general shape of $C(x)$.
3. Enter $C(x)$ into Maple as a function. Then plot $C(x)$. How does the Maple plot compare with your sketch?
4. Is $C(x)$ an even or odd function?
5. Evaluate the following by hand:

$$\frac{d}{dx}C(x) \tag{2}$$

$$\frac{d}{dx}5C(x) \tag{3}$$

$$\frac{d}{dx}C(x^3) \tag{4}$$

$$\frac{d}{dx}C(1/x) \tag{5}$$