Improper Integrals

Consider the following two functions:

$$f(x) = \frac{1}{x^2}, \quad g(x) = \frac{1}{\sqrt{x}}. \tag{1}$$

1. Evaluate the following definite integrals:

$\int_1^{10} f(x)\,dx$	
$\int_1^{100} f(x)\,dx$	
$\int_1^{10000} f(x)\,dx$	
$\int_1^{100000} f(x)\,dx$	

2. What happens to $\int_1^b f(x)\,dx$ as b gets larger and larger?
3. Now, evaluate these definite integrals:

$\int_1^{10} g(x)\,dx$	
$\int_1^{100} g(x)\,dx$	
$\int_1^{10000} g(x)\,dx$	
$\int_1^{100000} g(x)\,dx$	

4. What happens to $\int_1^b g(x)\,dx$ as b gets larger and larger?
5. Why is your answer to 4 different than your answer to 2? Try sketching $f(x)$ and $g(x)$.

Improper Integral Practice

Evaluate the following improper integrals.

1. $$\int_0^\infty 3e^{-4x}\,dx$$

2. $$\int_0^\infty xe^{-2x}\,dx$$

3. $$\int_0^8 \frac{dy}{y-4}$$

Further Impropriety

Again consider the following two functions:

$$f(x) = \frac{1}{x^2}, \quad g(x) = \frac{1}{\sqrt{x}}. \tag{2}$$

1. Evaluate the following definite integrals:

$\int_{0.1}^{1} f(x)\,dx$	
$\int_{0.01}^{1} f(x)\,dx$	
$\int_{0.001}^{1} f(x)\,dx$	

2. What happens to $\int_a^1 f(x)\,dx$ as a gets closer to zero?
3. Now, evaluate these definite integrals:

$\int_{0.1}^{1} g(x)\,dx$	
$\int_{0.01}^{1} g(x)\,dx$	
$\int_{.001}^{1} g(x)\,dx$	

4. What happens to $\int_1^b g(x)\,dx$ as b gets larger and larger?
5. Why is your answer to 4 different than your answer to 2? Try sketching $f(x)$ and $g(x)$.

Even More Improper Behavior

Now consider the following function:

$$f(x) \;=\; \sin(x)\;. \tag{3}$$

1. Evaluate the following definite integrals:

$\int_0^{100} f(x)\,dx$	
$\int_0^{1000} f(x)\,dx$	
$\int_0^{10000} f(x)\,dx$	
$\int_0^{10001} f(x)\,dx$	
$\int_0^{10002} f(x)\,dx$	

2. What happens to $\int_0^b f(x)\,dx$ as b gets larger and larger? What's going on here?
3. Without doing the integral, what can you say about

$$\int_0^{\infty} e^{-x}\sin(x)\,dx\;? \tag{4}$$

Does it exist? Is it finite?