Chapter 7.7: Improper Integrals
(What if the Cat Runs Infinitely Fast?)
Calculus II
Spring 2021
College of the Atlantic

Again consider the following two functions:

$$f(x) = \frac{1}{x^2}, \quad g(x) = \frac{1}{\sqrt{x}}. \tag{1}$$

1. Evaluate the following definite integrals:

$\int_{0.1}^{1} f(x)\,dx$	
$\int_{0.01}^{1} f(x)\,dx$	
$\int_{0.001}^{1} f(x)\,dx$	

2. What happens to $\int_{a}^{1} f(x)\,dx$ as a gets closer to zero?
3. Now, evaluate these definite integrals:

$\int_{0.1}^{1} g(x)\,dx$	
$\int_{0.01}^{1} g(x)\,dx$	
$\int_{.001}^{1} g(x)\,dx$	

4. What happens to $\int_{1}^{b} g(x)\,dx$ as b gets larger and larger?
5. Why is your answer to 4 different than your answer to 2? Try sketching $f(x)$ and $g(x)$.

More Improper Integrals

Now consider the following function:

$$f(x) = \sin(x) . \tag{2}$$

1. Evaluate the following definite integrals:

$\int_0^{100} f(x)\, dx$	
$\int_0^{1000} f(x)\, dx$	
$\int_0^{10000} f(x)\, dx$	
$\int_0^{10001} f(x)\, dx$	
$\int_0^{10002} f(x)\, dx$	

2. What happens to $\int_0^b f(x)\, dx$ as b gets larger and larger? What's going on here?