Class 01: Accumulated Change

Calculus II

College of the Atlantic. January 9, 2023

Figure 1: Unicorns with cupcakes and, surprisingly, a Rubik's cube. Image by rawpixel.com. Image source `https://www.rawpixel.com/image/515491/cute-unicorn-stickers`

1. Let $U(t)$ represent the total biomass of unicorns on an island, measured in units of kg, where t is the time, in months, since January 1, 2023. Values of $U(t)$ are shown in the table below

x	$U(x)$
0	463
2	578
4	602
6	612
8	582
10	631
12	689

   (a) What is the average rate of change of the unicorn biomass from month zero to month two? Be sure to put units on your answer.

   (b) What is the average rate of change of the unicorn biomass from month two to month four?

   (c) What is the formula for the average rate of change of $U(t)$ from $t = 0$ to $t = 2$?

   (d) What is the formula for the average rate of change of $U(t)$ from $t = a$ to $t = b$?

2. Now let $u(t)$ denote the *rate* at which the unicorn biomass is changing, in units of kg/month, where t is measured in months since January 1, 2023. Values for $u(t)$ are shown in the table below. This is a different island than the one in the previous problem. On this island, the biomass of unicorns on January 1 was 400 kg.

x	$u(x)$
0	30
2	40
4	55
6	55
8	60
10	70
12	75

(a) What is the biomass[1] of the unicorns after two months?
(b) What is the biomass of the unicorns after four months?
(c) What is the biomass of the unicorns after one year?
(d) Muse on the difference between your upper and lower estimates.

3. Soy milk is leaking from a storage tank in COA's dining hall. The rate of leakage is shown on the graph.

(a) Come up with upper and lower estimates for the total amount of soy milk that has been released into the environment. Use $\Delta t = 2$.
(b) Represent these upper and lower estimates on the graph.
(c) Show how you would represent upper and lower estimates using $\Delta t = 1$. Do not calculate numerical values for the estimates.
(d) Suppose you needed to know how much soy milk was released into the environment to within 4 gallons. What Δt would you choose?

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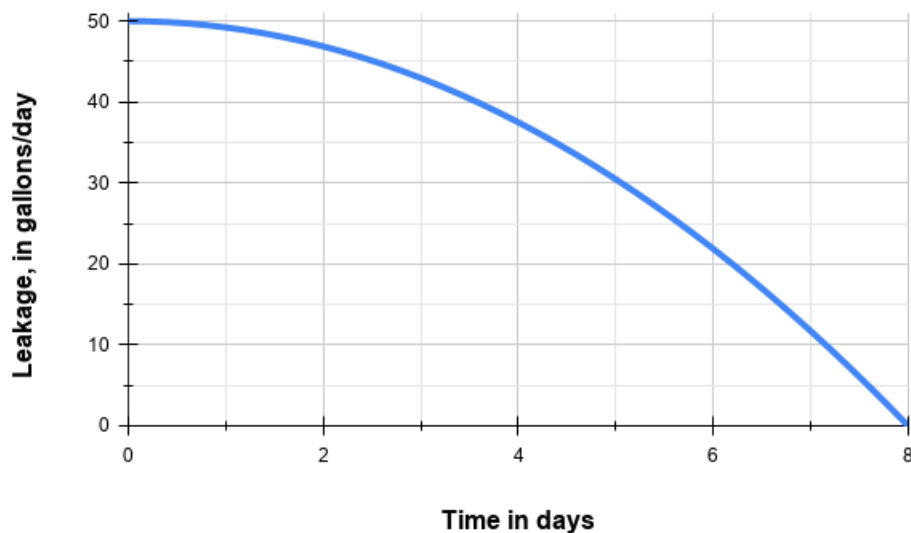
[1]You can't determine this exactly. (Why??) Instead, you can come up with an upper and lower estimate.