Class 07:
Short Summary of What We've Done So Far
Calculus II

College of the Atlantic. January 23, 2023

Left-Hand Sums and Right-Hand Sums

So far in Calculus II we have investigated how change accumulates. We have used left- and right-hand sums to calculate the total change: either the total change in unicorn biomass or the total amount of soy milk that gets spilled.

To do this calculation, we need to know the rate of change. We have considered three cases:

1. A **table** of values
2. Reading the rates off of a **graph** (this was the soy milk example.)
3. Using a **formula** for the rate.

In class on Thursday we worked did case three, working with the formula $u(t) = 50 + t^2$. First we did it by hand (with a calculator) for $\Delta t = 3$ and $\Delta t = 2$. Then in the afternoon class, we wrote some python code that would do this for any Δt.

The Sums Converge!

The left-hand sum is an over-estimate of the total accumulated change, and the right-hand sum is an under-estimate (or vice-versa). As Δt gets closer and closer to zero, the LHS and RHS converge—they approach the same value.

We saw this in class on Thursday afternoon. Both the left-hand and right-hand sums approached the number 1176.

The Definite Integral!!

Here is a compact way write this:

$$\int_0^{12} u(x)\,dx \;=\; 1176\;. \tag{1}$$

In words: "The definite integral of $u(x)$ from 0 to 12 is 1176."

The definite integral is the name for what happens to a left- or right-hand sum when $\Delta t \longrightarrow 0$.