Class 10: The Fundamental Theorem of Calculus Anti-Derivatives, Graphically and Analytically

Calculus II

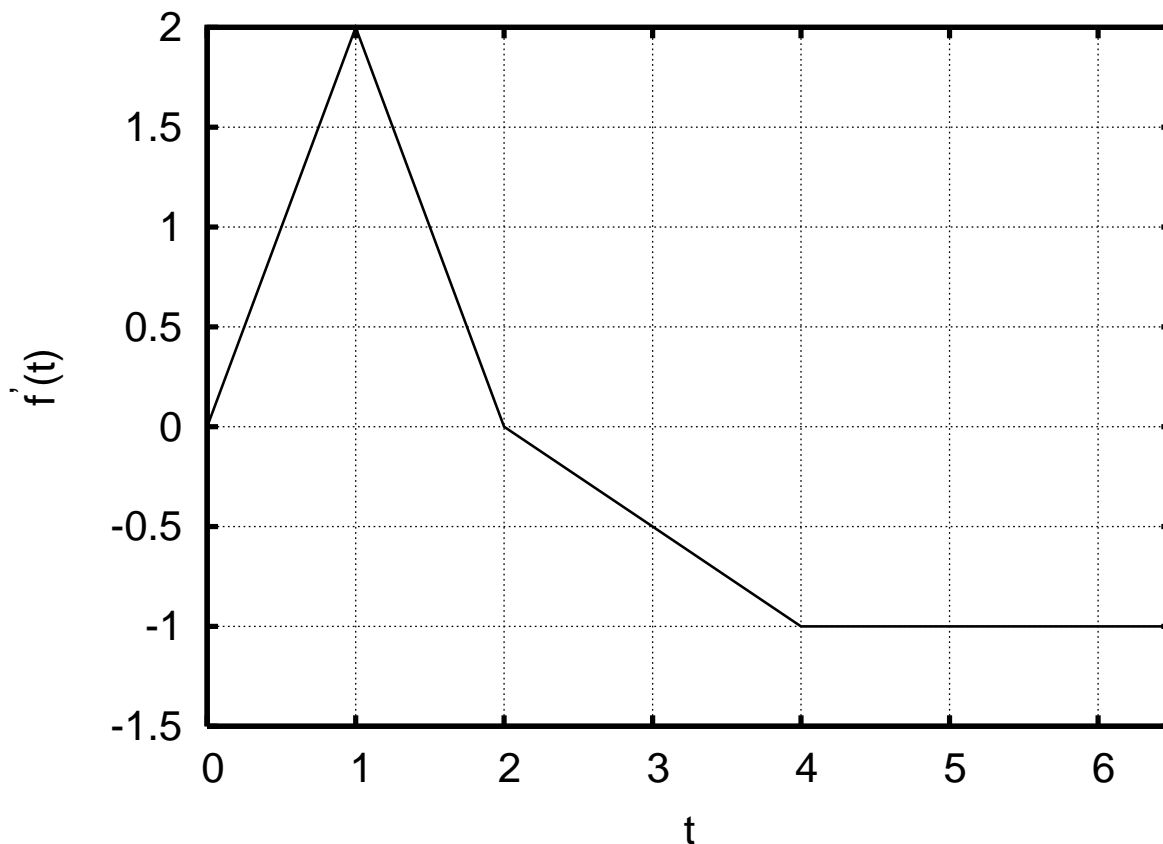**College of the Atlantic. January 30, 2023**

Figure 1: A plot of $f'(t)$, the rate, in thousands of gallons a day, at which water flows into a reservoir. A negative rate means that water is flowing out.

1. Determine $f(t)$, the water level in the reservoir as a function of time, given that at $t = 0$ the water level was 10. Enter your results in the table.

2. Determine $f(t)$, the water level in the reservoir as a function of time, given that at $t = 0$ the water level was 15. Enter your results in the table.

3. Plot the two $f(t)$'s that you found on the graph provided.

4. When is the level of water in the reservoir increasing at the fastest rate?

5. When is the level of water in the reservoir the greatest?

Time	Water Level	Water Level
0	10	15
1		
2		
3		
4		
5		
6		
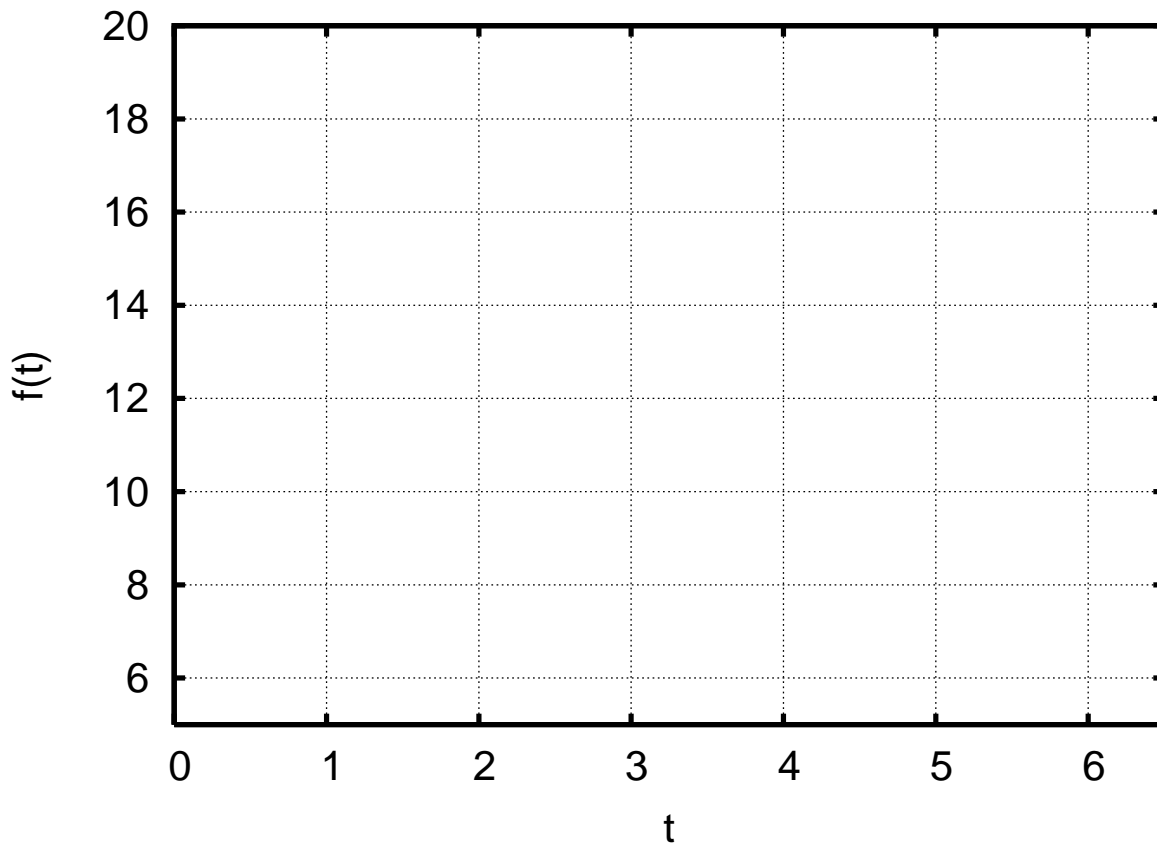
1. Take the **derivative** of the following functions:

   (a) $f(x) = \frac{1}{x}$
   (b) $f(x) = \sqrt{x} + 4x^{3/2}$
   (c) $f(x) = 2e^x$
   (d) $f(x) = 2^x$
   (e) $f(x) = 2\sin x$
   (f) $f(x) = \ln(x)$

2. Find the following **anti-derivatives**:

$$\int 4\,dx \tag{1}$$

$$\int 4x\,dx \tag{2}$$

$$\int \frac{4}{x}\,dx \tag{3}$$

$$\int (y + y^2 + y^3)\,dy \tag{4}$$

$$\int 2\sin(x)\,dx \tag{5}$$

$$\int 4t\,dt \tag{6}$$

$$\int \ln(x)\,dx \tag{7}$$

3. Find the following **definite integrals**:

$$\int_1^3 4x \, dx \tag{8}$$

$$\int_1^3 4t \, dt \tag{9}$$

$$\int_0^{\pi/2} \cos(x) \, dx \tag{10}$$

$$\int_0^{2\pi} \cos(x) \, dx \tag{11}$$

$$\int_{-2}^2 y^5 \, dy \tag{12}$$

4. What is the average value of $\cos(x)$ from $x = 0$ to $x = \pi/2$? Represent this average value graphically.