

Class 13: Arc Length and Density

Calculus II

College of the Atlantic. Feb 6, 2023

1. Use the arc length formula to calculate the arc length of $f(x) = (4/3)x + 2$ from $x = 3$ to $x = 6$. Explain why your answer is comforting. It's probably easiest if you resist the urge to convert any square roots you might encounter into decimals.
2. Let $f(x) = x^2$. How long is the curve from $x = 0$ to $x = 1$?
3. Let $f(x) = x^3$. How long is the curve from $x = 0$ to $x = 1$?
4. Let $f(x) = \sin(x)$. How long is the curve from $x = 0$ to $x = \pi$?
5. A cylindrical pipe has a radius of 5 cm and is 2 meters long.
 - (a) What is the volume of the pipe?
 - (b) If the pipe was filled with water, what would its mass be?
 - (c) Suppose there is some pollution in the water. The density of the pollution is given by $\rho(x) = 0.002e^{-2x}$, in units of grams per cubic meter. Determine the total mass of the pollution in the pipe.
6. The air density h meters above the surface of the earth is given by:

$$\rho(h) = 1.28e^{-0.000124h}, \quad (1)$$

where h has units of km and the units of ρ are kg/m^3 . What is the mass of a cylindrical column of air 2 meters in diameter and 25 kilometers high, with a base on the surface of the earth?