Class 17: Density

Calculus II

College of the Atlantic. Feb 16, 2023

1. A solid lead cylinder has a radius of 0.5 meters and a length of 3 meters. The density of lead is $11,340$ kg/m^3. What is the mass of the cylinder?

2. A certain type of plankton[1] likes to live on the bottom of the ocean. The density of the plankton decreases as the distance z from the bottom of the ocean increases. The density of the plankton is given by

$$\rho(z) = 10e^{-z}\,, \tag{1}$$

where ρ has units of kg/m^3, and z, the vertical distance up from the ocean floor, is measured in meters.

Consider a circular patch of ocean floor with a radius of 0.5 meters.

(a) What is the meaning of the statement $\rho(4) = 0.1831$?

(b) What is the total mass of the plankton in a 20 meter tall column of water above this patch of ocean floor?

(c) What is the total mass of the plankton in this water column between 5 and 15 meters above the ocean floor?

(d) What is the total mass of the plankton in the column of water *exactly* 17 meters above the ocean floor?

[1] Not actual biology. I'm pretty sure there aren't plankton like this. But this is a math class so it's ok.

3. The density of air on earth varies with altitude. The density is well approximated[2] by:

$$\rho(z) = 1.28e^{-\alpha z}\,, \tag{2}$$

where ρ has units of kg/m^3, and z, the vertical distance up from sea level, is measured in meters. The constant α has an approximate value of 0.00012.

Consider a circular patch of land at seal level with a radius of 0.5 meters.

(a) What are the units of α?

(b) What is the total mass of the air above the patch of land, contained in a column of air that is 1 km tall?

(c) What is the mass of the air that is *exactly* 314 meters above the patch of land?

(d) What is the total mass of the air above the patch of land, contained in a column of air that is 10 km tall?

(e) What is the total mass of *all* the air above the patch of land?

[2]Actual physics. Sorta. This equation assumes that the temperature is constant, which is an ok approximation for smallish ranges of z. In Thermodynamics one of the homework problems is deriving this equation.