Class 19: Improper Integrals:
Eternal Soy Milk Leakage
Calculus II
College of the Atlantic. Feb 20, 2023

Figure 1: Photo of soybeans and soymilk. Image by wikipedia user Kjokkenutstyr, available under Create Commons License. Image source: https://commons.wikimedia.org/wiki/File:004-soymilk.jpg. See also https://www.kjokkenutstyr.net/.

1. Soymilk is leaking from a storage container at a rate given by $r(t) = 1/t^2$, where t is measured in days, and $r(t)$ is measured in tons per day. The leak begins on day $t = 1$.

   (a) How much soy milk leaks from day 1 to day 10?
   (b) How much soy milk leaks from day 1 to day 100?
   (c) How much soy milk leaks from day 1 to day 1000?
   (d) The leak goes on for eternity. How much soy milk leaks out during this time?

2. There is another soymilk leak, again starting at day $t = 1$. The rate at which the soymilk leaks is now given by $r(t) = 1/t$.

   (a) How much soy milk leaks from day 1 to day 10?
   (b) How much soy milk leaks from day 1 to day 100?
   (c) How much soy milk leaks from day 1 to day 1000?
   (d) The leak goes on for eternity. How much soy milk leaks out during this time?

3. In what is becoming a worrying trend, there is yet another soymilk leak, again starting at day $t = 1$. This time rate at which the soymilk leaks is given by $r(t) = e^{-t}$.

   (a) How much soy milk leaks from day 1 to day 10?
   (b) How much soy milk leaks from day 1 to day 100?
   (c) How much soy milk leaks from day 1 to day 1000?
   (d) The leak goes on for eternity. How much soy milk leaks out during this time?

4. There is another soymilk leak. This is by now not surprising, but is certainly still cause for concern. In any event, in this case the rate of leakage is given by $r(t) = e^{-t} \cos t$. The leak begins at $t = 1$.

   (a) Over the course of eternity, does a finite or infinite amount of soy milk leak out of the container? You should be able to confidently answer this question without using a computer or pencil-and-paper to evaluate any integrals.