Course Summary

Calculus II

College of the Atlantic. Winter 2023

1. **Big Ideas**:

   (a) Things change, and we can define an *instantaneous* rate of change. This is the **derivative**.

   (b) Change accumulates. We can add up accumulated change, even if the instantaneous changes are changing while the accumulation is happening. This is the **definite integral**.

2. Integral Definitions and Fundamentals

   (a) By how much did the unicorn population change? What is the total amount of soy milk that leaked out?
   (b) Definition of definite as limit of left- and right-hand sums, Riemann sums.
   (c) Using python to construct and evaluate LH- and RH-sums.
   (d) Definite integral as area under a curve.
   (e) First fundamental theorem: total change of function equals integral of little changes:

$$\int_a^b f(x)\,dx \;=\; F(b) - F(a)\;, \quad \text{where } F'(x) = f(x)\;. \tag{1}$$

   We use this theorem to evaluate definite integrals without having to do sums. It is convenient when it's convenient.
   (f) Constructing antiderivatives graphically and numerically
   (g) Second fundamental theorem:

$$\frac{d}{dx}\int_a^x f(z)\,dz \;=\; f(x) \tag{2}$$

3. Integration Techniques

   (a) Guess and check
   (b) u substitutions
   (c) ~~Integration by parts~~
   (d) WolframAlpha
   (e) Improper integrals (what if the limit(s) of a definite integral are infinite?).

4. Integration Applications

   (a) Areas and volumes, volumes of revolution
   (b) Arc length
   (c) Density
   (d) Probability density and cumulative density functions
   (e) Central limit theorem, distribution of a sampling mean.
   (f) z-scores, z-tables, p-values

5. Sequences and series
    (a) Geometric series
    (b) Convergence of series
    (c) Convergence tests: comparison, limit comparison, alternating, ratio
    (d) Power series
    (e) Taylor series

Calculus II Course Axioms

Winter 2023

In mathematics, axioms are propositions that are assumed to be true. The mathematician Federico Ardila-Mantilla has written four axioms that guide the work he does in education and outreach. Federico's axioms resonate strongly with me. They are:

1. Mathematical potential is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.
2. Everyone can have joyful, meaningful, and empowering mathematical experiences.
3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
4. Every student deserves to be treated with dignity and respect.

Calculus II Course Goals

Winter 2023

1. Stay physically and mentally healthy and maintain intellectual and personal connection.
2. Experience the challenge, joy, and beauty of calculus and mathematics in general.
3. Improve your problem solving skills and mathematical confidence. Leave this course with an increased ability to do mathematics.
4. Gain a firm, grounded, enduring understanding of one of the big ideas of calculus: the integral.
5. Gain a good introduction to infinite sum and series.
6. Gain experience using some basic programming in python to help learn mathematics.
7. Improve your skills at communicating problem solving strategies in writing, both for yourself and for others.
8. Have fun while learning a lot.