Class 01: Accumulated Change

Calculus II

College of the Atlantic. January 6, 2025

Figure 1: Unicorns with cupcakes and, surprisingly, a Rubik's cube. Image by rawpixel.com. Image source
`https://www.rawpixel.com/image/515491/cute-unicorn-stickers`

1. A remote island has a population of unicorns. Let $u(t)$ denote the *rate* at which the unicorn biomass is changing, in units of kg/month, where t is measured in months since January 1, 2025. Values for $u(t)$ are shown in the table below.

t	$u(t)$
0	30
2	40
4	55
6	55
8	60
10	70
12	75

(a) By how much has the unicorn biomass changed[1] of the unicorns after two months?
(b) By how much has the unicorn biomass changed after four months?
(c) By how much has the unicorn biomass changed after one year?
(d) Muse on the difference between your upper and lower estimates.
(e) The biomass of unicorns on January 1, 2025 was 400 kg. What is the unicorn biomass on January 1, 2026?
(f) If the biomass of unicorns on January 1, 2025 was 1000 kg. What is the unicorn biomass on January 1, 2026?

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[1]You can't determine this exactly. (Why??) Instead, you can come up with an upper and lower estimate.

2. At dinnertime, whoever is cooking dinner rings the dinner bell when food is ready. A unicorn hears the bell, and after a short delay runs to get dinner. The unicorn runs at a velocity of 10 m/s for 4 seconds. How far has the unicorn run?

3. A different unicorn hears the bell, waits, and then starts running quickly and then slows down. The unicorn's velocity is given by $v(t)$, where v is measured in m/s, and t is measured in seconds since the bell rang. Values for $v(t)$ are shown in the table below:

t	$v(t)$
2	20
4	15
6	10
8	5
10	0

   (a) How far has the unicorn run? As before, you can't figure this out exactly.

   (b) Or wait, maybe you *can* figure it out exactly?

4. A different unicorn hears the dinner bell and runs to dinner. Its velocity is given by the $v(t)$ values shown below.

t	$v(t)$
6	20
8	15
10	10
12	5
14	0

   (a) How far has this unicorn run?

5. Yet another unicorn hears the dinner bell and runs to dinner. Its velocity is given by the $v(t)$ values shown below.

t	$v(t)$
2	40
4	23
6	10
8	2.5
10	0

   (a) How far has this unicorn run?

   (b) Is it possible to figure out the distance exactly? If not, what would you need to get a better estimate of the distance the unicorn has ran?