Class 07: Accumulated Change: More with Graphs

Calculus II

College of the Atlantic. January 20, 2025

1. Let $r(t)$ be the rate, in people per minute, at which people arrive at the dining hall for dinner, where t is measured in minutes past 5:30. Consider the following integral:

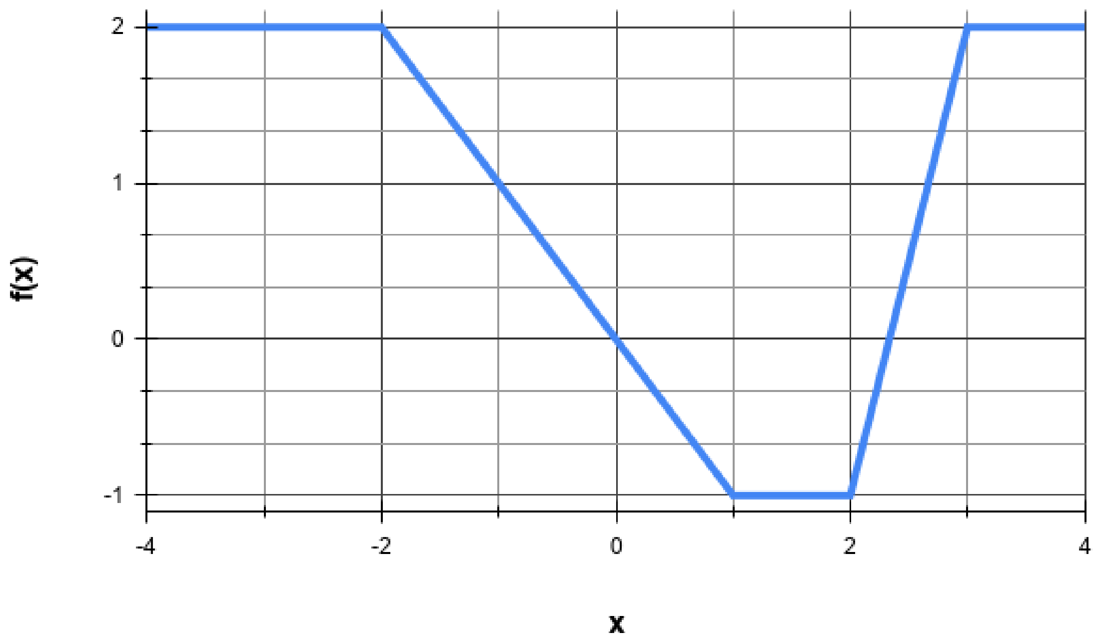
$$\int_0^{30} r(t)\, dt\ . \tag{1}$$

   (a) What are the units of the above integral?

   (b) What is the practical interpretation of the above integral?

   (c) What are the units of $\frac{dr}{dt}$?

   (d) What is the practical interpretation of $\frac{dr}{dt}$?



2. A function $f(x)$ is shown above. Note the location of the vertical zero axis. Use the graph to determine values of the following:

   (a) $\int_{-4}^{-2} f(x)\, dx$

   (b) $\int_{-2}^{0} f(x)\, dx$

   (c) $\int_{-4}^{0} f(x)\, dx$

   (d) $\int_{0}^{2} f(x)\, dx$

   (e) $\int_{2}^{3} f(x)\, dx$

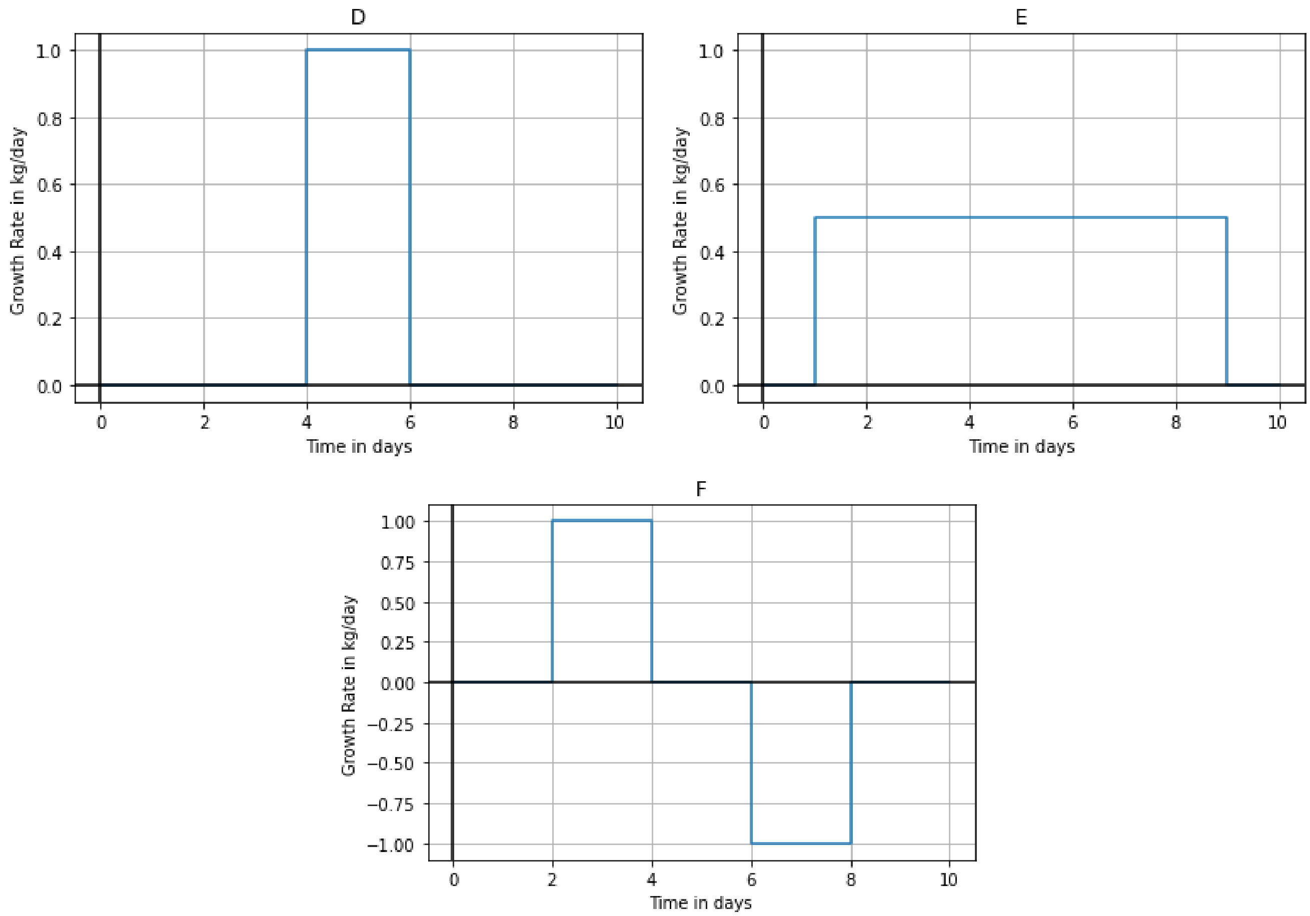


Figure 1: Three different rates of change of the biomass of unicorns.

3. The plots in Fig. 1 show three different possible functions for $u(x)$, the rate of change of unicorn biomass, in units of kg/day.

   (a) For each $u(x)$ make a quantitatively accurate sketch of $U(t)$, the total unicorn biomass as a function of time. Assume that $U(0) = 3$. (Evidently these are very small unicorns.)

   (b) Repeat the above problem, but assume that $U(0) = 4$.

4. Let $u(t)$ represent the rate of change, in kg/day, of the biomass of unicorns on an island. The time t is measured in days since Jan 1, 2025.

   (a) What do the following quantities represent in practical terms?

$$\int_0^{10} u(t)\, dt \ . \tag{2}$$

$$\int_0^{10} u(z)\, dz \ . \tag{3}$$

$$\int_{10}^{20} u(t)\, dt \ . \tag{4}$$

$$\int_0^{10} u(t)\, dt \ . \tag{5}$$

   (b) Now consider the following function of t:

$$U(t) \ = \ \int_0^{t} u(z)\, dz \ . \tag{6}$$

      i. What does $U(t)$ represent in practical terms? What are its units?

      ii. What is the meaning of $\frac{d}{dt}U(t)$? Muse on this for a while.

5. Let $F(t)$ represent the total change, given a rate of change $f(t)$. That is:

$$F(t) \ = \ \int_0^{t} f(z)\, dz \ , \tag{7}$$

   Where $f(t)$ is the function plotted on the first page of this handout. Make a reasonably accurate sketch of $F(t)$. Assume that $F(0) = 1$.