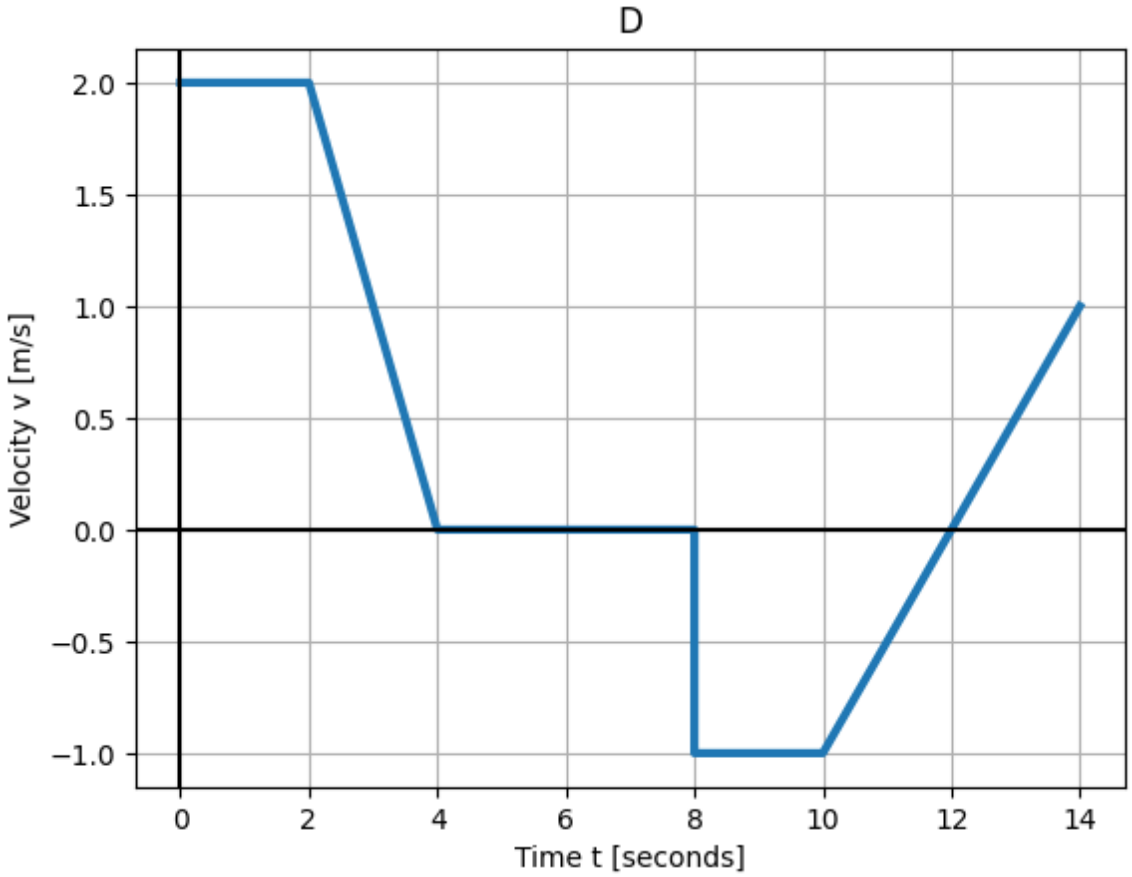
Class 08: The Fundamental Theorems of Calculus

Calculus II

College of the Atlantic. January 22, 2025



1. The plot above shows the velocity of a unicorn as a function of time t. Let $x(t)$ denote the positon of the unicorn as a function of time, and let's assume that $x(0) = 5$.

   (a) Make a reasonably accurate sketch of $x(t)$.

   (b) Write a formula for $x(t)$. Your formula will involve a definite integral and $x(0)$.

2. Below are some amount functions. For each function, find the corresponding rate function:

$$F(t) = 100 . \tag{1}$$

$$F(t) = 100 + 5t + t^2 . \tag{2}$$

$$F(t) = e^t + t^2 . \tag{3}$$

$$F(t) = \sin(t) . \tag{4}$$

3. Below are some rate functions. For each function, find the corresponding amount function:

$$f(t) = 30 . \tag{5}$$

$$f(t) = 10 + t . \tag{6}$$

$$f(t) = 4t^2 + 7t^3 . \tag{7}$$

$$f(t) = e^t . \tag{8}$$

$$f(t) = \sin(t) . \tag{9}$$

4. Evaluate the following definite integrals. Do the answers make sense?

$$\int_0^{10} 5\,dt\ . \tag{10}$$

$$\int_0^{10} 5t\,dt\ . \tag{11}$$

$$\int_{-10}^{10} 5t^3\,dt\ . \tag{12}$$

$$\int_0^{2} e^t\,dt\ . \tag{13}$$

$$\int_0^{2} e^x\,dx\ . \tag{14}$$

$$\int_0^{\pi} \sin(t)\,dt\ . \tag{15}$$

$$\int_0^{2\pi} \sin(t)\,dt\ . \tag{16}$$