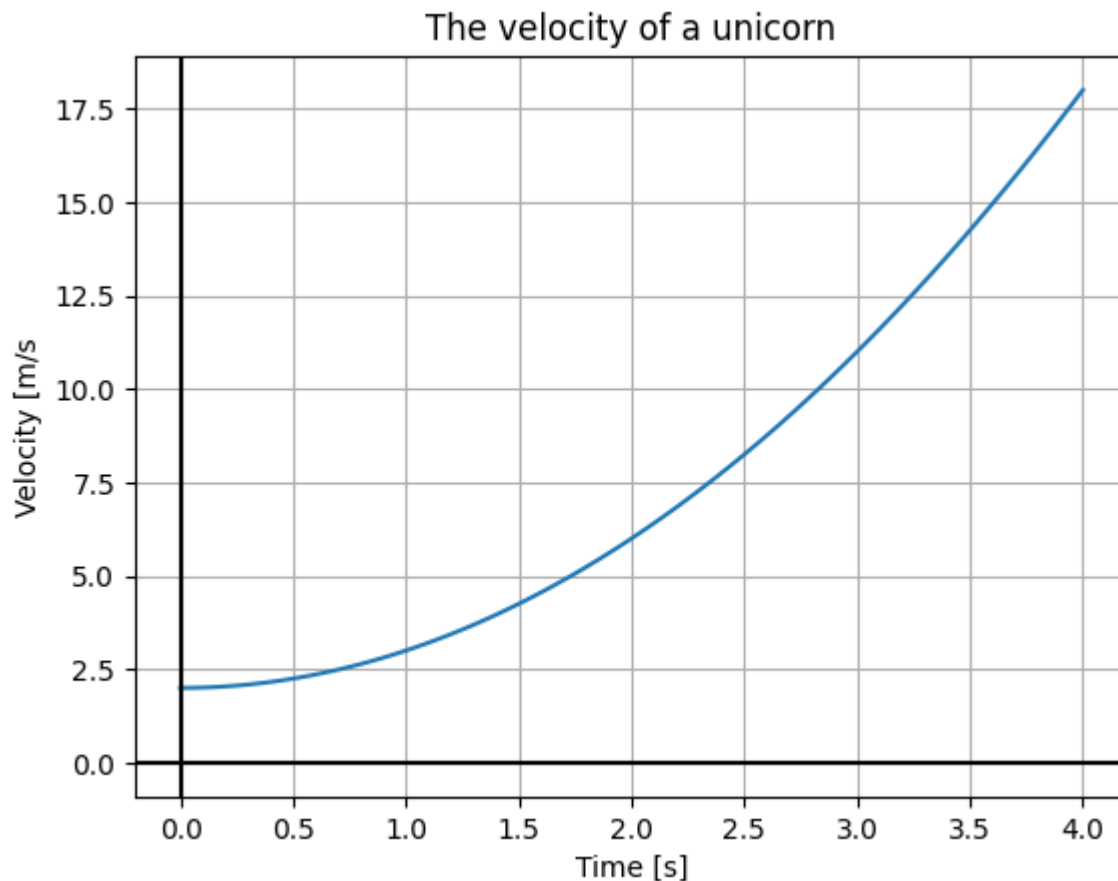


Class 09: Averages of Functions

Calculus II

College of the Atlantic. January 23, 2025



1. The plot above shows the velocity of a unicorn as a function of time t . Let $x(t)$ denote the position of the unicorn as a function of time.
 - (a) How far has the unicorn run from time $t = 0$ to $t = 5$? Use the graph to come up with a rough estimate.
 - (b) On the graph, show how to represent the distance the unicorn travels from $t = 0$ to $t = 5$.
 - (c) Write the distance traveled as a definite integral.
 - (d) What do you think is the equation of $v(t)$ shown in the figure?
 - (e) Use your answer to the above question, and the fundamental theorem of calculus, to determine an exact number for the distance the unicorn has traveled.
 - (f) Use your answer to the above question to figure out the average velocity of the unicorn from $t = 0$ to $t = 5$?
 - (g) Show how to illustrate this average velocity on the graph of $v(t)$.

2. On a weird Maine day, the temperature is described by the following function: $T(t) = 25 + \frac{1}{4}t^2$, where time t is measured in hours since midnight.
- (a) What is the average temperature that day – i.e. over the next 24 hours.
 - (b) Sketch $T(t)$.
 - (c) On your sketch, show how to represent the average temperature.
3. Try answering these by thinking, instead of actually evaluating a definite integral:
- (a) What is the average value of $f(x) = 3$ from $x = -2$ to $x = 2$?
 - (b) What is the average value of $f(x) = x$ from $x = -2$ to $x = 2$?
 - (c) What is the average value of $g(x) = \cos(x)$ from $x = 0$ to $x = 2\pi$?
 - (d) What is the average value of $h(x) = 1 + \cos(x)$ from $x = 0$ to $x = 2\pi$?
4. Here is a rough description of the stages of a rocket launch:
- The engine ignites. Lift-off!
 - The engine burns, exerting a constant upward force on the rocket. The rocket accelerates upwards.
 - The rocket engine burns out, so there is no more thrust. The rocket “coasts” upward, slowing down and turning around. (Picture what happens to a ball that you throw upward, after the ball has left your hand.)
 - A little explosive inside the rocket goes off, the rocket opens up, and a parachute opens. The rocket falls to earth.

Make a rough, qualitative sketch of a possible $z(t)$ (altitude z as a function of time) for this scenario.