Class 17: Density Calculus II

College of the Atlantic. Feb 12, 2025

1. A solid lead cylinder has a radius of 0.5 meters and a length of 3 meters. The density of lead is $11,340 \text{ kg/m}^3$. What is the mass of the cylinder?

2. A certain type of plankton¹ likes to live on the bottom of the ocean. The density of the plankton decreases as the distance z from the bottom of the ocean increases. The density of the plankton is given by

$$\rho(z) = 10e^{-z} \,, \tag{1}$$

where ρ has units of kg/m³, and z, the vertical distance up from the ocean floor, is measured in meters.

Consider a circular patch of ocean floor with a radius of 0.5 meters.

- (a) What is the meaning of the statement $\rho(4) = 0.1831$?
- (b) What is the total mass of the plankton in a 20 meter tall column of water above this patch of ocean floor?
- (c) What is the total mass of the plankton in this water column between 5 and 15 meters above the ocean floor?
- (d) What is the total mass of the plankton in the column of water *exactly* 17 meters above the ocean floor?

¹Not actual biology. I'm pretty sure there aren't plankton like this. But this is a math class so it's ok.