Class 19: More Probability Density:
Means and the Normal Distribution
Calculus II

College of the Atlantic. Feb 24, 2025

Figure 1: Left: A unicorn with wings. Image by brgfx on Freepik. Right: Mini marshmallows. Image by wikipedia contributor Dvortygirl. Image source: wikipedia.licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license.

Unicorns are created when a cosmic ray with an energy greater than 50 TeV (Tera Electron Volts) interacts with a marshmallow. See figures above. An international team of scientists has created a unicorn generator device by densely packing a large room with marshmallows.

Cosmic rays hit the earth at a constant probabilistic rate. In any hour, the probability that cosmic ray with a sufficient energy to create a unicorn hits the marshmallow unicorn generator, is 0.03. The waiting time between unicorn creation events is a random variable t that is distributed according to:

$$\rho(t) = \lambda e^{-\lambda t}, \tag{1}$$

Where $\lambda = 0.03$, in units of probability per hour, and t is measured in hours.

At exactly 8:00am you observe a unicorn creation event. After seeing this event:

1. How long would you have to wait so there is a 50% chance that you see another unicorn created?

2. What is the average (mean) waiting time between events?

After several months of using this method to produce unicorns, you notice that the mass of the unicorns are not all the same. You take some measurements, do some math, and conclude that the distribution of unicorn masses is well approximated by:

$$\rho(x) = \frac{1}{10\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-200}{10}\right)^2}, \tag{2}$$

where x is the unicorn mass measured in kilograms.

1. Sketch $\rho(x)$. Use a computer if you want, but be sure to understand why the distribution looks the way it does.

2. What is the probability that a randomly-chosen unicorn has a mass between 190 and 210?

3. What is the probability that a randomly-chosen unicorn has a mass of 200?

4. What is the probability that a randomly-chosen unicorn has a mass greater than 200?

5. What is the probability that a randomly-chosen unicorn has a mass between 0 and infinity?