Calc II Rocket Assignment 2: Drag and Terminal Velocity

Due March 7, 2025

- If you want, you can do this assignment in pairs and submit only one write-up.
- We'll start (and finish?) this in class today (Wednesday 5 March).
- For all problems, **please show your work**. Thanks.

Drag is a force that acts to oppose the motion of a moving object. The direction of the drag force is always opposite to the direction of motion. We'll start by thinking about drag qualitatively.

1. Actually, let's start by not considering drag at all. Suppose a ball is dropped from a height of 50 meters. Ignoring drag, the ball accelerates downward at a constant rate of -9.8m/s^2.

   (a) Sketch the position, velocity, and acceleration of the ball.

   (b) Determine a formula for $v(t)$, the ball's velocity as a function of time.

   (c) Determine a formula for $z(t)$, the ball's altitude as a function of time.

2. Now we'll think about dropping a ball, but this time we won't ignore drag.

   (a) Draw a free-body diagram for the ball at following times:

      i. Immediately after the ball is dropped.

      ii. A few seconds after the ball is dropped but before the ball has reached terminal velocity.

      iii. After the ball has reached terminal velocity.

   (b) Sketch the position, velocity, and acceleration of the ball.

3. The magnitude of the drag force on an object moving at speed v is given by:

$$F_{\text{drag}} = \frac{1}{2}\rho A C_d v^2 \ , \tag{1}$$

where ρ is the density of air (or whatever), A is the cross-sectional area of the object, and C_d is the drag coefficient. We can group all these constants together and write Eq. (1) as

$$F_{\text{drag}} = cv^2 \ , \tag{2}$$

where c is a constant. At terminal velocity v_t, the drag force equals the force of gravity, so the net force is zero. (The force of gravity is mg, where m is the mass of the ball.

   (a) Convert the previous sentence into an equation, and solve that equation for c.

   (b) Now write this equation another way: solve for v_t^2.

   (c) Suppose we measured the terminal velocity for balls of different masses, and then plotted v_t^2 versus m. The plot should be a line. (why?) What is the slope of the line?

4. So we now know how to experimentally calculate c. Let's now try to figure out $v(t)$, the ball's velocity as a function of time. For free-fall, we did this in problem 1b and it was kinda easy. How is it different if we account for drag? Let's see. We'll start with Newton's second law:

$$F_{\text{net}} = ma \ . \tag{3}$$

   (a) Write down Newton's second law for the situation: falling with drag. Let the downward direction be positive.

   (b) The acceleration is just the rate of change of velocity; $a = \frac{dv}{dt}$. Plug this in to your previous answer.

   (c) We know the rate of change of v. Why can't we just use an accumulator function (a definite integral) to calculate the total change in v, as we've been doing throughout the class?