

16.1: Integrals in One and Two Dimensions

Calculus III

College of the Atlantic

- Roadkill occurs along a 100-mile stretch of road. The Department of Transportation has determined that the rate of occurrence is given in the table below, in units of roadkill per mile per month.
 - Find upper and lower estimates for the number of animals killed each month on this road.
 - Suppose you knew that the roadkill density was described by the function $\rho(x)$. Write an exact expression for the total number of roadkill per month.
 - Write an approximate expression for the average density of roadkill along this stretch of road.
 - Write an exact expression for the average density of roadkill along this road in terms of the density function $\rho(x)$.
- In a region of the UK, someone has compiled remarkably detailed information about the spatial distribution of foxes. The map in Fig. 1 shows contour lines for fox density, measured in foxes per square kilometer. Estimates for the density in each square on the map are found in Fig. 2. These estimates were formed by using the approximate density value in the northeast corner of each box.
 - Write an approximate expression for the total number of foxes.
 - Suppose that the spatial density of foxes is known to be given by the function $\sigma(x, y)$, in units of foxes per square kilometer. Write down an exact expression for the total number of foxes.
 - Write down an exact expression for the average fox density.
- Estimate $\int_A x^2 y \, dA$ where A is the rectangle whose corners are: $(0, 0)$, $(2, 0)$, $(0, 1)$, and $(2, 1)$. Do not use the fundamental theorem of calculus. Instead, make a table of numbers with $\Delta x = 0.5$ and $\Delta y = 0.5$.
- The following problems concern a rectangular region with corners $(0, 0)$, $(2, 0)$, $(0, 3)$, and $(2, 3)$.
 - Write down an exact expression for the area between the $z = 0$ plane and the function $f(x, y) = x^2 + y^2$ on the rectangular region.
 - Write down an exact expression for average value of the function $f(x, y) = x^2 + y^2$ on the rectangle.
 - Using an integral, write down an expression for the area of the rectangle.
- Express the above integrals as iterated integrals, and evaluate them.

Mile	Roadkill Density
0	8
25	4
50	3
75	3
100	0

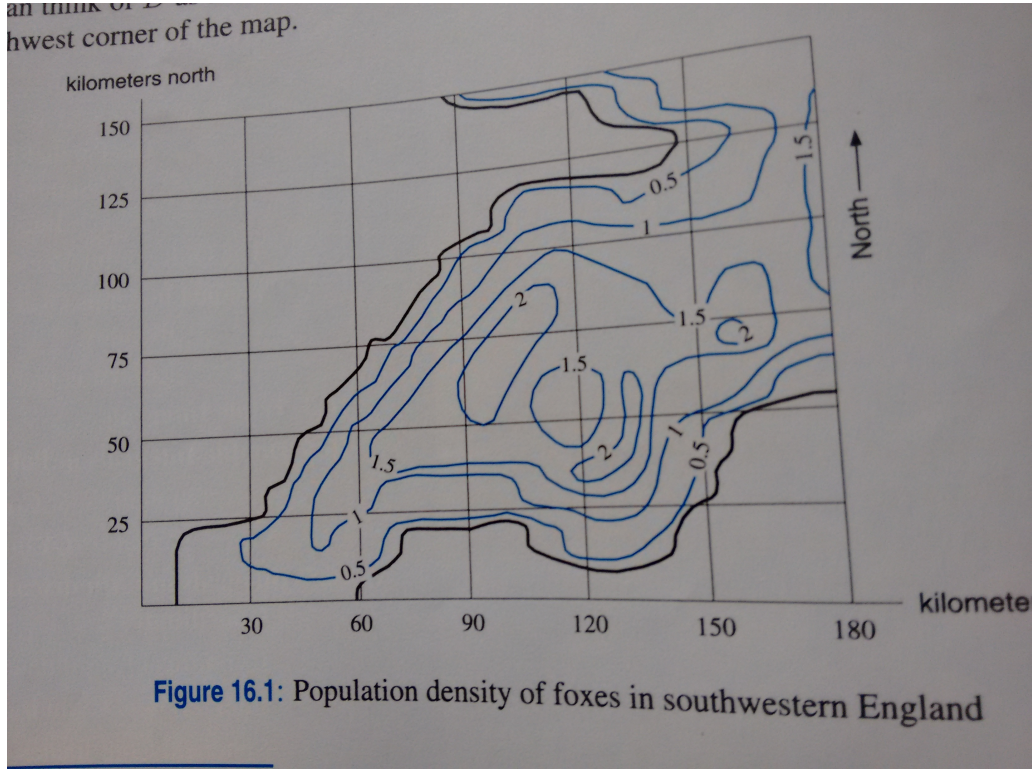


Figure 1: A contour map of fox densities.

16.1 THE DEFINITE INTEGRAL OF A FUNCTION OF TWO VARIABLES

Table 16.1 Estimates of population density (northeast corner)

0.0	0.0	0.2	0.7	1.2	1.2
0.0	0.0	0.0	0.0	0.1	1.6
0.0	0.0	0.5	1.4	1.1	1.6
0.0	0.0	1.5	1.8	1.5	1.3
0.0	1.1	2.0	1.4	1.0	0.0
0.0	1.0	0.6	1.2	0.0	0.0

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Figure 2: A table of fox densities.