19.2: Evaluating Flux Integrals

Calculus III

College of the Atlantic

1. Let $\vec{F} = (y+1)\vec{i} + x\vec{j}$. Find a scalar function f such that $\nabla f = \vec{F}$.

2. Consider the vector field:
$$\vec{F} = 3\vec{i} - 4\vec{j} + 5\vec{k} \, . \tag{1}$$
Consider a cube with one corner at the origin. The length of each side of the cube is 2, extending in the positive direction. I.e., the coordinates of the corners are $(0,0,0)$, $(0,0,2)$, $(0,2,0)$, $(0,2,2)$, $(2,0,0)$, $(2,0,2)$, $(2,2,0)$, $(2,2,2)$. Determine the flux out of the cube:

   (a) Calculate the flux through the top of the cube.

   (b) Calculate the flux through the bottom of the cube.

   (c) Calculate the flux through the left of the cube.

   (d) Calculate the flux through the right of the cube.

   (e) Calculate the flux through the front of the cube.

   (f) Calculate the flux through the back of the cube.

3. Repeat the above question using the following field:
$$\vec{G} = z\vec{i} - 3\vec{j} + y\vec{k} \, . \tag{2}$$

4. Repeat the above question using the following field:
$$\vec{H} = z\vec{i} - 3y\vec{j} + y\vec{k} \, . \tag{3}$$

5. Consider the vector field:
$$\vec{E} = kq\frac{\vec{r}}{r^3} \, . \tag{4}$$
This describes the electric field due to a charge q at the origin.

   (a) Compute the flux of $\vec{F}$ out of a sphere of radius 2 centered at the origin.

   (b) Compute the flux of $\vec{F}$ out of a sphere of radius 3 centered at the origin.

   (c) What do you conjecture would be the flux of $\vec{F}$ out of a cube of side 2 centered at the origin? Why?