

## 20.1: Divergence

### Calculus III

College of the Atlantic

1. The density of air in a room is given by  $f(x, y, z) = z + xy^2$ . What is the mass of air in a sphere of radius 0.01 centered at  $(1, 2, 3)$ ?
2. A vector field has a divergence  $\nabla \cdot \vec{F} = z + xy^2$  in  $\text{m}^3/\text{s}$ . What is the flux flowing out of a sphere of radius 0.01 centered at  $(1, 2, 3)$ ?
3. Find the divergence of the following vector fields:

$$\vec{F} = y\hat{i} - x\hat{j}, \quad (1)$$

$$\vec{G} = x\hat{i} + y\hat{j}, \quad (2)$$

$$\vec{H} = x\hat{i} - y\hat{j}, \quad (3)$$

$$\vec{Q} = y\hat{i} - x^2y\hat{j} + x\hat{k}, \quad (4)$$

4. Consider the vector field  $\vec{F} = 5y\hat{j}$ .
  - (a) Use the geometric definition to find the divergence at  $(0, 0, 0)$ . Use a cube of side  $c$  centered at  $(0, 0, 0)$ .
  - (b) Use the geometric definition to find the divergence at  $(3, 3, 0)$ . Use a cube of side  $c$  centered at  $(3, 3, 0)$ .
  - (c) Compute the divergence of  $\vec{F}$  and confirm that it is consistent with your answers above.