20.3: Curl

Calculus III

College of the Atlantic

We will work with the following vector fields:

$$\vec{E} = x\hat{\imath} + y\hat{\jmath} \ , \tag{1}$$

$$\vec{B} = y\hat{\imath} - x\hat{\jmath} \ , \tag{2}$$

$$\vec{F} = -2\hat{\imath} \ , \tag{3}$$

$$\vec{G} = -(y+1)\hat{\imath} \ , \tag{4}$$

0. Let $\vec{a} = 3\hat{\imath}$ and $\vec{b} = 2\hat{\imath} + 2\hat{\jmath}$. Calculate $\vec{a} \times \vec{b}$ using both the geometric and algebraic definitions of the cross product.
1. Sketch each of the above vector fields.
2. Calculate the circulation around a circle of radius a centered at the origin for $\vec{E}$ and $\vec{B}$.
3. Calculate the circulation around a square of side s centered at the origin for $\vec{F}$ and $\vec{G}$.
4. Use your circulation calculations to determine the circulation density of each of the fields.
5. Calculate the curl of each of the four fields using the algebraic definition of curl.
6. Which of the fields have zero (positive, negative) divergence? Answer using both geometric and algebraic arguments.
7. For each of the four fields, use Stokes' Theorem to calculate $\oint_C \vec{F} \cdot d\vec{r}$, where C is a clockwise square of side 2 centered at $(5,5)$. Do your answers depend on the position of the center of the square? Do your answers depend on the side of the square?
8. Repeat the above question, but use the field $\vec{H} = -(y^2+1)\hat{\imath}$.

9. Let $\vec{F}(x, y, z)$ be a vector field and $f(x, y, z)$ is a scalar function of three variables. Which of the following quantities are vectors and which are scalars. Which are not defined?

(a) $\operatorname{div} \vec{F}$

(b) $\operatorname{curl} \vec{F}$

(c) $\operatorname{div} f$

(d) $\operatorname{curl} f$

(e) ∇f

(f) $\nabla \vec{F}$

(g) $\nabla \times \vec{F}$

(h) $\nabla \cdot \vec{F}$

(i) $\nabla \cdot f$

(j) $\nabla \times f$

(k) $\nabla \cdot \nabla f$

(l) $\nabla \times \nabla f$

(m) $\nabla \cdot \nabla \times \vec{F}$