Functions of two variables

On a separate sheet of paper is a numerical representation of a function of two variables. Let's call the two variables x and y and we'll call the function $f(x, y)$.

1. What would a plot of this function look like? Make a rough sketch?
2. At $x = 1$ and $y = 1$, is the function increasing in the x direction? Is it increasing in the y direction?
3. At $x = -1$ and $y = 1$, is the function increasing in the x direction? Is it increasing in the y direction?
4. At $x = 1$ and $y = -1$, is the function increasing in the x direction? Is it increasing in the y direction?
5. At $x = -1$ and $y = -1$, is the function increasing in the x direction? Is it increasing in the y direction?
6. Consider all the x, y pairs for which $f(x, y) = 1$. What does the set these x, y pair look like? Make a sketch.
7. Consider all the x, y pairs for which $f(x, y) = 2$. What does the set these x, y pair look like? Make a sketch.
8. Consider all the x, y pairs for which $f(x, y) = 3$. What does the set these x, y pair look like? Make a sketch.
9. Let's suppose that x measures the number of hours of sleep you get, measured above or below your average amount. I.e., if $x = 0.5$, that means you got half an hour of sleep more than average. And let y measure the temperature of the room in which you sleep, measured in degrees above or below the average temperature.[1] The function $f(x, y)$ measures your unhappiness, in arbitrary units. The picture here, I guess, is that you have a set sleeping routine and don't like any disruption. Sketch and interpret, in words, the following:

   (a) $f(1, y)$

[1] Ok. I realize this isn't a very good example of a function. But it was hard to come up with something that seemed realistic given that I had decided to use both positive and negative x and y values.

(b) $f(2, y)$

(c) $f(x, -1)$

(d) $f(x, 0)$

10. By staring at the numbers, guess a formula for the function $f(x, y)$.
11. Using this formula, come up with algebraic answers to questions 6–8.
12. Using this formula, come up with algebraic answers to question 9.