# Functions of Two Variables

## Calculus III

### College of the Atlantic. Winter 2016

On a separate sheet of paper is a numerical representation of a function of two variables. Let's call the two variables $x$ and $y$ and we'll call the function $f(x, y)$.

1. What would a plot of this function look like? Make a rough sketch.

2. At $x = 1$ and $y = 1$, is the function increasing in the $x$ direction? Is it increasing in the $y$ direction?

3. At $x = -1$ and $y = 1$, is the function increasing in the $x$ direction? Is it increasing in the $y$ direction?

4. At $x = 1$ and $y = -1$, is the function increasing in the $x$ direction? Is it increasing in the $y$ direction?

5. At $x = -1$ and $y = -1$, is the function increasing in the $x$ direction? Is it increasing in the $y$ direction?

6. Consider all the $x, y$ pairs for which $f(x, y) = 1$. What does the set of these $x, y$ pairs look like? Make a sketch.

7. Consider all the $x, y$ pairs for which $f(x, y) = 2$. What does the set of these $x, y$ pairs look like? Make a sketch.

8. Consider all the $x, y$ pairs for which $f(x, y) = 3$. What does the set of these $x, y$ pairs look like? Make a sketch.

9. Let's suppose that $x$ measures the number of hours of sleep you get, measured above or below your average amount. I.e., if $x = 0.5$, that means you got half an hour of sleep more than average. And let $y$ measure the temperature of the room in which you sleep, measured in degrees above or below the average temperature.[1] The function $f(x, y)$ measures your unhappiness, in arbitrary units. The picture here, I guess, is that you have a set sleeping routine and don't like any disruption. Sketch and interpret, in words, the following:

   (a) $f(1, y)$
   (b) $f(2, y)$
   (c) $f(x, -1)$
   (d) $f(x, 0)$

10. What is the meaning of the following quantities:

   (a) $f(0, 0)$
   (b) $f(2, -1)$

11. By staring at the numbers, guess a formula for the function $f(x, y)$.

12. Using this formula, come up with algebraic answers to questions 6–8.

13. Using this formula, come up with algebraic answers to question 9.

**Distances and Spheres:**

1. How far is the point $(3, 4, 5)$ from the origin?

2. How far is the point $(3, 4, 5)$ from the point $(1, 2, 3)$?

3. Write down the equation of a sphere with radius 5 centered at the origin.

4. Write down the equation of a sphere with radius 5 centered at the point $(2, -2, 5)$.

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[1]Ok. I realize this isn't a very good example of a function. But it was hard to come up with something that seemed realistic given that I had decided to use both positive and negative $x$ and $y$ values.